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# What is the Natural Weight of the Current Old ? \*

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**Abstract** We consider a simple overlapping generations model with an externality *à la* Arrow-Romer. A government with the power to tax wishes to maximize the utility of the agents alive in period 0, and possibly that of their children. Is there a natural relative weight for the utility of the current old ? We content that there is and show it is not 1.

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# 1 Introduction

The overlapping generations model of Allais (1947), Samuelson (1958) and Diamond (1965) is ideally suited to the exploration of inter-generational issues. For a more recent and thorough presentation of these models, see De la Croix and Michel (2001).

Most models assume that agents live for two periods hence an objective measure of time is inherent to the model. The length of one period is usually taken to be 20 or 30 years; the training of the young may or may not be explicitly modelled. Temporary equilibria and intertemporal equilibria are analyzed and a role for government intervention appears naturally if market imperfections such as externalities are present; public finance issues can also be considered.

Here, contrary to most of the literature on overlapping generations models and growth we do not focus on either the stationary equilibrium or the welfare of all generations from the present onwards. Our argument is that if government intervention is warranted, the proposed policies should be acceptable to people who are alive at the time. Policies that optimize over the very long run but lower the welfare of the current generation — compared with the status quo of no intervention at all — have little chance of being adopted. Hu (1979, p. 283) was well aware of this point. Therefore the government criterion will be the welfare of people alive in period 0.

We begin with a standard Diamond-like model with an externality of the learning-by-doing type (Arrow (1962), Sheshinski (1967) and Romer (1986)). The aggregate stock of capital has a beneficial effect on the efficiency of production by each firm. This externality can be internalized through government intervention in the form of fiscal policies.

## 2 The Individuals

Individuals live for two periods: in the first period they consume, save and inelastically supply one unit of labor. In the second period they live off the revenue from their savings. There is no population growth,  $L_t = L_{t-1} = L$ .  $c_t$  and  $d_t$  are the consumptions of young and old, respectively, in period  $t$ ;  $s_t$  is savings,  $(1 - \theta_t)w_t$  is the net wage rate,  $\theta_t$  is the rate of tax on wage income and  $R_t$  is the rent of capital.  $\beta$  is the discount factor. An individual born in period  $t$  solves the following programme:

$$\max_{c_t, d_{t+1}} u(c_t) + \beta u(d_{t+1}) \quad (1)$$

$$\text{subject to } c_t + s_t = (1 - \theta_t)w_t \quad (2)$$

$$d_{t+1} = R_{t+1}s_t \quad (3)$$

The optimality condition is

$$u'(c_t) = \beta R_{t+1}u'(d_{t+1}) \quad (4)$$

### 3 The Firm

The production function exhibits constant returns to scale to the factors hired by the firm but there is an externality. The firm hires  $l_t$  units of labor and produces  $q_t$  units of good with  $k_t$  units of capital;  $B(K_t)$  represents the externality where  $K_t$  is the total stock of capital

$$q_t = B(K_t)F(k_t, l_t) \quad (5)$$

where  $F$  is homogeneous of degree one.

$$q_t = l_t B(K_t) f(k_t/l_t) \quad (6)$$

Normalizing the size of the firm at  $l_t = 1$ , we have:

$$q_t = B(K_t)f(k_t) = B(Lk_t)f(k_t) \quad (7)$$

The firm maximizes profit

$$\pi(k_t) = B(Lk_t)f(k_t) - w_t - (R_t - \tau_t)k_t \quad (8)$$

where  $\tau_t$  is a government subsidy designed to internalize the externality. Therefore

$$R_t = \tau_t + B(Lk_t)f'(k_t) \quad (9)$$

$$w_t = B(Lk_t) [f(k_t) - f'(k_t)k_t] \quad (10)$$

An informed government would choose

$$\tau_t = LB'(Lk_t)f(k_t) \quad (11)$$

Capital depreciates entirely in one period, hence the dynamics of the economy are given by

$$k_{t+1} = s_t \quad (12)$$

## 4 The Government

The government budget constraint for any fiscal scheme  $(\theta_t, \tau_t)$  is, for each period,

$$\theta w_t = \tau_t k_t \quad (13)$$

Therefore we can express one of the taxes in terms of the other.

The government objective is to maximize the welfare of people who are alive at the time. Therefore, it takes into account the utility of the old and young in period zero, plus the utility of those who will be old in period one. In the existing literature all individuals are given the same weight — except for the discount factor. Sometimes the utility of the young born in period  $-1$  (hence those who are old in period 0) is also included, but treated as exogenous, therefore irrelevant. See for instance De la Croix and Michel (2001), p. 91. This seems like a natural assumption for individuals who live for two periods. However a single period is included for the old of period 0. Therefore we introduce a ponderation of their utility in the government objective <sup>1</sup>.

The analysis proceeds as follows: First, find a tax scheme that satisfies the budget constraint (13) and maximizes the government welfare criterion

$$W = \eta u(d_0) + u(c_0) + \beta u(d_1) \quad (14)$$

This tax system depends on the ponderation  $\eta$  of the old. Secondly use the efficient subsidy from equation (11) and the government budget constraint in (13) to characterize  $\theta_t$ . Rationalizing both results yields a condition on  $\eta$ .

It is the contention of this note that this weight is naturally different from 1. In order to make our argument simple and concise we now turn to a special case of the model.

## 5 A Simple Case

We consider the case of logarithmic utility and Cobb-Douglas production. Therefore conditions (4), (7), (9), and (10) become, respectively,

$$d_{t+1} = \beta R_{t+1} c_t \quad (15)$$

$$q_t = L^\sigma k_t^\sigma k_t^\alpha \quad (16)$$

where  $0 \leq \sigma \leq 1$  measures the strength of the externality.

$$R_t = \tau_t + \alpha L^\sigma k_t^{\sigma+\alpha-1} \quad (17)$$

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<sup>1</sup>They have contributed the current stock of capital by their savings in the previous period.

$$w_t = (1 - \alpha)L^\sigma k_t^{\sigma+\alpha} \quad (18)$$

The government objective is to maximize welfare function, i.e.:

$$W = \eta \ln(d_0) + \ln(c_0) + \beta \ln(d_1) \quad (19)$$

We use (13) to eliminate  $\tau_t$  from our calculations and easily obtain

$$d_0 = [\alpha + (1 - \alpha)\theta_0] L^\sigma k_0^{\alpha+\sigma} \quad (20)$$

$$c_0 = (1 - \theta_0)(1 - \alpha)L^\sigma k_0^{\alpha+\sigma} \quad (21)$$

$$d_1 = \beta R_1 c_0$$

$$d_1 = \frac{\beta R_1}{1 + \beta} (1 - \theta_0)(1 - \alpha)L^\sigma k_0^{\alpha+\sigma} \quad (22)$$

with

$$R_1 = [\alpha + (1 - \alpha)\theta_1] L^\sigma \left[ \frac{\beta}{1 + \beta} (1 - \alpha)(1 - \theta_0)L^\sigma k_0^{\alpha+\sigma} \right]^{\alpha+\sigma-1} \quad (23)$$

Irrespective of the length of the horizon adopted by the government as its guiding criterion it is inevitable that the last  $\theta$  value included,  $\theta_1$  here, will only tax the young of the last period, whose utility is not included in the criterion. It appears as the expected return on savings of the previous generation; therefore the highest value would be selected. This is clearly unreasonable and a dynamic structure must be imposed on  $\theta_t$ . (This problem is avoided when the horizon is infinite and there is no last period.) Here we choose the simplest form:  $\theta_t = \theta, \forall t$ . (We shall see later that this is a wise choice.)

The first step is to characterize the  $\theta$  value that maximizes (19):

$$\frac{(\beta + \eta)(1 - \alpha)}{\alpha + (1 - \alpha)\theta} = \frac{1 + \beta(\alpha + \sigma)}{1 - \theta} \quad (24)$$

Therefore the welfare-maximizing  $\theta$  value is

$$\theta^* = \frac{\beta [1 - \alpha(1 + \alpha + \sigma)] - \alpha(1 + \eta)}{(1 - \alpha) [1 + \beta(\alpha + \sigma) + \beta + \eta]} \quad (25)$$

This welfare-maximizing tax depends on the parameters of the problem, including the weight of the old generation  $\eta$ , which has so far been exogenously selected.

Secondly, we do have additional information on the correct choice of  $\theta$ . The efficient value of  $\tau_t$  that internalizes the externality is (from (11) and (16))

$$\tau_t = \sigma L^\sigma k_t^{\sigma+\alpha-1} \quad (26)$$

When combined with the government budget constraint (13) and (18) we obtain

$$\theta_t = \frac{\sigma}{1 - \alpha} \quad (27)$$

Therefore the efficient choice of  $\theta$  is known and it is also constant — as foreshadowed. Combining (27) and (25) we find that there is only one value of  $\eta$  that is consistent with both. It depends on the strength of the externality and we denote it by  $\eta_\sigma$  :

$$\eta_\sigma = \frac{(\alpha + \sigma)(1 + (\alpha + \sigma)\beta)}{1 - \alpha - \sigma} - \beta \quad (28)$$

In the special case of no externality, there is no need for government intervention, as (27) makes clear, and the “natural”  $\eta$  value is

$$\eta_0 = \frac{\alpha(1 + \alpha\beta)}{1 - \alpha} - \beta \quad (29)$$

The expression in (29) is always less than 1 for sensible values of  $\alpha \in [0, 1/2]$ . There is a similar result for (28) but  $\alpha + \sigma \in [0, 1/2]$  cannot be assumed. However, the assumption of  $\eta = 1$  is obviously unjustified, even in one of the simplest versions of an overlapping generations model.

The expressions in (28) and (29) clearly depend on the features of the model as well as on the length of the horizon selected by the government. Calculations in cases when  $W$  also includes terms such as  $\ln(c_1)$  and  $\ln(d_2)$  yield expressions similar to (28) and (29). Another type of production function such as  $q_t = Ak_t + B_t l_t$ , with  $B_t = L^\sigma k_t^\sigma$  also yield simple results.

## 6 Conclusion

We have shown that the “natural” value of the weight of the current old, that is, the value of  $\eta$  that reconciles the maximization of the chosen welfare function with the use of the efficient externality-correcting tax is less than 1.

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