TEPP WORKING PAPER

N° 2010 - 19

Bankruptcy Risk, Product Market Competition and Horizontal Mergers

BERNARD FRANCK, NICOLAS LE PAPE

www.tepp.eu

TEPP - Institute for Labor Studies and Public Policies
TEPP - Travail, Emploi et Politiques Publiques - FR CNRS 3126

ISSN 2110-5472

ISSN 2110-5472

Bankruptcy Risk, Product Market Competition and Horizontal Mergers

Bernard Franck* and Nicolas Le Pape†‡

July 2010

Abstract

We consider an oligopolistic industry including leveraged firms and unleveraged ones where firms are engaged in a sequential decision-making process. At the first stage of the game, a firm and her bank, considering the demand uncertainty and the distribution probability of the shock, evaluate a bankruptcy risk and, at the second stage, firms are engaged in a Cournot competition. We characterize subgame perfect equilibria and we analyze the impact on these equilibria of the proportion of debt financed firms in the industry. By introducing an additionnal upstream stage to the game, we then examine how debt financing impacts on incentives to merge with competitors. We demonstrate that a merger involving leveraged firms increases the bankruptcy probability of the merging firms while a similar merger concerning unleveraged firms or between the two categories of firms leads to a decrease in the bankruptcy probability of leveraged firms. Moreover, the minimum number of firms that must be engaged in the coalition in order to cause a profitable merger, is lower in comparison with Salant, Switzer and Reynolds (1983). The welfare losses associated to anticompetitive effects of mergers are lower when the coalition gathers unleveraged firms rather than leveraged ones. Our model predicts that in evaluating proposed mergers Competition Authorities should take into account the financial structure of both merging firms and outsiders. Moreover the model justifies the failing firm argument when an unleveraged firm takes over a leveraged one.

*Université de Caen, CREM. *Mail:* bernard.franck@unicaen.fr

†Université du Mans, GAINS-TEPP and CREM. *Mail:* nicolas.lepape@univ-lemans.fr

‡*Corresponding author:* Nicolas Le Pape, Faculté de Droit et des Sciences Economiques, Université du Maine, Avenue O. Messiaen, 72 085 Le Mans Cedex, FRANCE.

JEL Classification: G33; G34; L13; L41.

Keywords: Debt, Bankruptcy, Horizontal Merger,
Competition Policy, Oligopoly.

1 Introduction

From the middle of 1980's, a growing attention has been given to the interactions between a firm's financial structure and product market decisions[1]. Under Cournot-type quantity competition a firm is all the more aggressive towards its competitor than she is indebted (see for instance Brander and Lewis, 1986; Glazer 1994, Showalter, 1995). This result refers to the commitment value of the debt to subsequent output strategies and may be explained in the following way: because shareholders benefit from the limited liability when they run a leveraged firm[2], they choose a level of production without taking into account potential losses realized in case of default. Consequently, raising debt induces firms to pursue more aggressive output strategies because it increases profits in solvency states. A risky debt contract favors the conduct of aggressive product market strategies and intensifies the degree of competition. In such a context, the Modigliani-Miller neutrality result does not hold. The use of strategic debt has been studied under price or quantity competition, in the presence of demand or cost uncertainty: Showalter (1995) shows that, under price competition, firms enhance their position in the product market when a random shock affects demand but when costs are random firms may choose not use any strategic debt.

A key feature of the literature on strategic debt is that the connection between a firm's financial structure and its behaviour in the product market is established in a duopoly setting. As a consequence, the strategic value of the debt is not examined in a more general oligopoly context where competing firms may merge so that the number of firms may vary. The impact of the strategic use of the debt on the firm's aggressiveness is not recognized as an incentive to merge in order to decrease the intensity of competition. When non cooperative firms promote aggressive strategies it makes way for horizontal mergers in order to attenuate the product market competition. Actually and at the firm level, the benefit from a decrease in the intensity of competition by using an horizontal merger is connected with the degree of competition which prevails before the merger. The benefits associated with the anti-competitive effects of mergers are *a priori* all the more important as firms before the merger were engaged in an aggressive stance. Consequently, when we take into consideration the pro-competitive effect of the debt, the financial structure of firms becomes a key determinant in the private profitability of horizontal mergers.

In this paper, we depart from the literature in Industrial Organization devoted to the strategic affect of financial decisions by generalizing the analysis of Brander and Lewis (1986) to an oligopolistic industry including leveraged firms and unleveraged ones and by considering the incentive to merge with competitors when some firms bear a bankruptcy risk. We want to capture the unilateral effects of mergers when firms finance their production with issuing debt. Our objective is to examine to what extend the private profitability and welfare con-

[1]For a survey of the literature on the relation between corporate finance and product market competition see for instance Tirole (2006, Ch. 7) or Maksimovic (1999).

[2]For finance to play a role in product market competition, we need to consider that shareholders control the firm strategy.

sequences of mergers are modified when some competitors bear a bankruptcy risk. We also evaluate the level of the risk of failure in a context where firms may be engaged in horizontal mergers.

Nevertheless, the literature dealing with the issue of anticompetitive effects of horizontal mergers does not highlight the impact of financial structure on the individual production decisions. In a Cournot oligopoly setting with symmetric non indebted firms, if we assume that mergers do not generate efficiency gains, it is well known that most of horizontal mergers are unprofitable[3] unless a large proportion of rival firms are engaged in the merger: the private incentive to merge appears only if at least 80% of firms are merging (Salant, Switzer and Reynolds, 1983). This "merger paradox" (minimum market share that the merging parties require in order to merge profitably) is traditionally relaxed by adopting a Bertrand competition with differentiated products (Deneckere and Davidson, 1985), by assuming firms merging creates a leader (Daughety, 1990 or Levin, 1990) or by recognizing increasing marginal costs (Perry and Porter, 1985). An increase in the profitability of mergers also appears when two stage games are considered and in particular in models where the managerial contract acts as a commitment variable. When the production decisions are delegated to managers, the product market competition is exacerbated (Fershtman and Judd, 1987 or Sklivas, 1987) and as result the profitability of horizontal mergers increases (Ziss, 2001, Gonzales-Maestre and Lopez-Cunat, 2001, Krakel and Sliwka, 2006). As far as we know, the question of the impact of the indebtedness in a game with horizontal mergers has been addressed in only two articles. Kanatas and Qi (2004) focuses on the impact of outstanding debt in the financial structure of a high cost firm to motivate takeovers by a healthy and low cost competitor. They exhibit conditions under which a leveraged firm does not choose to exit the market and motivates the healthy firm to acquire it. Nevertheless, we largely differ from their analysis since they assume no bankruptcy risk and since they consider that the debt has no strategic role on product market decisions (the firm output is independent of the level of the debt, the key role of limited liability status on production strategies is not introduced). Socorro (2007) considers a Cournot oligopoly with demand uncertainty and shows that the collusive impact of horizontal merger is mitigated by the financing mode of firms : debt financing implies that merging parties compete more aggressively due to limited liability. However, contrary to our model, the level of the debt is exogenously given so that it is not modified by the merger of firms.

Using a two-stage model, we consider production decisions of leveraged firms that are vulnerable to bankruptcy. We construct a model of Cournot competition where firms produce a homogeneous output and sell it on a market with uncertainty over demand. We evaluate the expected bankruptcy risk and the optimal leverage in relation with the number of firms getting into debt. The debt contract involve an estimate (by the firm and by the bank) of the expected risk of failure since the firm faces a random demand which is distributed ac-

[3] Private incentive to merge is highly dependent on the nature of competition. If one considers a Bertrand oligopoly with differentiated product, mergers are always profitable (Deneckere and Davidson, 1985).

cording to a given density function. The choice of the levels of the debt and of the quantities produced determines the threshold of the realization of the shock above which the firm is solvent. The distribution function of the shock is common knowledge which allows to evaluate an expected bankruptcy risk at the first stage of the game. Because of limited liability, leveraged firms have incentives to implement product market strategies which increase returns in the sole solvent states. Thus, when firms resort to debt, it distorts the firm's value optimization program at the output market stage. Debt financing induces shareholders, protected by the limited liability, to promote aggressive production strategies. We then examine consequences of horizontal mergers according to the financial structure of firms by introducing an additional upstream stage in the game: initially, firms decide to merge or not with rivals. As a consequence, the model takes into account the benefit of anticompetitive horizontal mergers when leveraged firms bear a bankruptcy risk . We evaluate the profitability of mergers between leveraged firms and the same profitability when the mergers concern unleveraged firms.

The antitrust policy on the control of concentrations also justify our model. Indeed, the competition laws for authorizing a merger allow for the level of bankruptcy risk of firms involved in the merger to weigh against anticompetitive consequences of the increasing concentration of the industry: the US anti-trust law contains the failing firm defense which permits to authorize mergers even if they strengthen a dominant position[4]. The loss of competition is not merger-specific if one of the merging parties is failing[5]. The European Commission develops the concept of a rescue merger: a merger would not be blocked (due to its effect on competition) if the firm to be acquired would in the near future be forced out of the market. Consequently, if the firm exits the market because of bankruptcy, the increase in market concentration (as a consequence of a reduction in the number of competitors) occurs independently of the horizontal merger. When the merger constitutes an alternative to the exit of the market by failure, the anti-trust authorities are more prone to allow the merger.

In the literature, there is a lack of articles dealing with the bankruptcy risk of firms implied in horizontal mergers : *"While the literature on mergers generally is very large, there are very few papers analyzing the failing firm defence specifically"* (Mason et Weeds, 2003, p.4) or *"little attention has been paid to failing firm rules in economic theory despite their policy relevance"* (Persson, 2005, p.177). According to Mason and Weeds (2003), competition authorities should be accommodating when they implement the failing firm argument insofar as the exit of the market by merger prior to the effective bankruptcy could give rise to increasing entry. Conversely, Fedele and Tognoni (2005) with a model of

[4] It was first applied in 1930 when the company International Shoe had acquired its competitor (McElwain) in financial distress.

[5] More precisely, the implementation of the failing firm argument rests upon three main criteria : (i) the failing firm in any event must be forced out of the market, (i.i) there is no alternative buyer who could provide for a less anti-competitive solution, (i.i.i) the market share of the acquired (failing) firm or its assets would be taken over by the acquiring firm or in the absence of merger its assets would exit the market.

horizontal mergers in a Cournot oligopoly with free entry show that a merger with a failing firm generates an additional production capacity which allows to deter the entry of new competitors. Persson (2005) analyzes the welfare consequences of the failing firm defence in a context where assets of the failing firm are sold. He identifies circumstances under which this acquisition rule the failing firm (no less anti-competitive alternative purchase) may lead to total surplus losses. These losses occur when smaller and inefficient firms are favored in the bidding process. We depart from these models by showing that welfare losses in relation with anti-competitive horizontal mergers are lower when a firm which eventually may default is acquired by a firm with no bankruptcy risk rather than by a firm which bears a positive bankruptcy risk.

In this paper, we demonstrate that mergers involving leveraged firms increase the bankruptcy probability of the merging firms while similar mergers concerning unleveraged firms or between the two categories of firms reduce the bankruptcy probability of leveraged firms. The welfare losses associated to anti-competitive effects of mergers are lower when the coalition gathers unleveraged firms rather than leveraged ones. The model justifies the failing firm argument when an unleveraged firm takes over a leveraged one.

The remainder of the article is organized as follows. In section 2 the model of strategic financial and operating choices is presented. Section 3 discusses the properties of product market equilibria when the number of firms is given. Private incentives to merge according to the bankruptcy risk of competitive firms are examined in section 4. Section 5 summarizes main results of the article and derives implications for competition policy.

2 A simple model of Cournot competition with bankruptcy risk

2.1 Hypotheses

We consider n firms that produce an homogeneous product and engage in quantity competition. All firms have the same marginal cost which is normalized to zero without loss of generality. In the industry, we distinguish two categories of firms according to their financial choice. A first group gathers n_1 firms which raise debt and bankruptcy occurs when the firm cannot fulfill its debt payments. A second group aggregates n_2 $(= n - n_1)$ firms which only use equity capital as a financing mode and consequently have no bankruptcy risk. These two categories of firms in competition on the product market may reflect differences in attitudes of entrepreneurs towards financing on equity capital. One could imagine that some firms do not want to finance on equity capital owing to the loss of control resulting from capital opening to outside shareholders[6]. Even though

[6] This kind of argument concerns the case of family firms for instance.

debt in financial structure creates a possibility of incurring a bankruptcy risk, it allows to keep the control on the running of the firm as well (at least on the states of the nature where the firm is able to meet its debts) . Conversely, firms choosing not to borrow money avoid such a transfer of control rights on firm's assets when insolvent states occur. Moreover, in the perspective of the literature on credit rationing (Stiglitz and Weiss, 1981) the lender adjust the level of the loan to the value of the collateral so that the category of unleveraged firms also includes firms ruled out of bank finance.

The production level of a leveraged [unleveraged] firm is q_i [q_j]. Operating profits are denoted by R^i [R^j] and are dependent on individual levels of production and on an additive random shock z on market price reflecting uncertain demand on the product market. The random variable z is assumed to be uniformly distributed over the interval $[-\overline{z}, +\overline{z}]$ with a density function $f(z) = 1/2\overline{z}$.

The inverse demand is given by:

$$p = a - Q + z$$

where Q represents the aggregate production.

Since the firm i has to repay its debt from its operating profit, she declares bankruptcy if its profit falls short its debt payment. There is thus a threshold value of z, noticed $\widehat{z_i}$, such that if the realized state of the world is such as $z < \widehat{z_i}$ the firm is bankrupt and if $z \geq \widehat{z_i}$, the firm is solvent and able to reimburse its debt. Consequently, equity holders are residual claimants only in the good states of nature $(z \geq \widehat{z_i})$

D_i is the level of firm's i short term debt (reimbursed on only one production game) and r_i is the interest rate. The level of repayments in the good states of nature is $B_i = (1 + r_i)D_i$ resulting in a threshold value $\widehat{z_i}$ defined by the condition : $R^i(q_i, q_j, \widehat{z_i}) = B_i$.

The cumulative distribution function F defines the bankruptcy probability $F(\ \widehat{z_i}) = \theta_i$ with:

$$\theta_i = \int_{-\overline{z}}^{\widehat{z_i}} f(z)dz = 1 - \int_{\widehat{z_i}}^{\overline{z}} f(z)dz$$

The model is a three-stage oligopoly game. Initially (stage 1), shareholders decide to merge or not taking into account the impact of merging decisions on bankruptcy risk and production choices. The exact number of competitors is given at this stage. In stage 2, for the category of firms with positive debt levels, each firm and her bank agree upon an expected bankruptcy risk[7]. This risk refers to the threshold value $\widehat{z_i}$ that maximizes the ex ante total value (equity and debt) of the firm . Finally, the firms make production decisions and the amount of the loan is calibrated to the estimated bankruptcy probability (stage 3). Shareholders choose an output market strategy that maximizes the

[7] Nevertheless, the study of mergers can also be done while keeping the original Brander and Lewis (1986) assumptions, but computation is then much heavier, since the equilibrium production of a firm is a solution to a second degree equation (see Franck and Le Pape, 2008).

equity value. All choices are made before the realization of the shock and payoffs are received when uncertainty on the demand is resolved. To obtain a subgame perfect equilibrium, the game is solved backwards.

2.2 Bankruptcy risk valuation and debt/product market strategies

We want to capture the impact of bankruptcy risk on subsequent product market strategies. This risk results from the realization of a demand random shock which is assumed to be common knowledge. The firm is bankrupt if the realization of the shock is below the threshold value ($\widehat{z_i}$), the bankruptcy probability is given by the cumulative distribution function F of z. We consider that the threshold value $\widehat{z_i}$, corresponding to the state of nature for which operating profit is just large enough to repay debt obligation, is given when output and debt decisions take place. In this case, firms are not allowed to change it the bankruptcy probability before production decisions take place. The model takes into account the fact that the bankruptcy risk is strategically used in order to take advantage in the product market[8]. Consequently, our approach contrasts with the standard literature on debt/product market interaction since models in the vein of Brander and Lewis (1986) recognize the debt obligation as a strategic device.

Three main economic arguments could be put forward in order to justify the commitment value of bankruptcy risk.

In the first place, during the 1990's, the context of disintermediation and of an increasing competition between banks led to banking strategies to prospect the fringe of customers endowed with a higher level of risk of failure which caused a growing proportion of the riskier borrowers in the credit portfolios of banking institutions. In such a context of the weakening of the financial stability of the banking industry, the Basel Committee (2001) on banking supervision has imposed that the granting of a loan must be contingent to an estimation of the debtor's default risk. Numerous models of the estimate of credit risk can be quoted: model CreditMetric of JP Morgan, model Credit Risk+ of the Crédit Suisse and so on... Using scoring models, the bank attributes a note to potential borrowers reflecting their expected probability of default and then she decides conditions on the loan (in particular the interest rate). Moreover, banks are induced to generalize such practises since those which develop the most accurate measure and management of credit risk will be imposed to lowest statutory needs in equity capital (risk-sensitive capital requirements). Consequently, these aforementioned transformations in the banking landscape justify the sequence of decisions retained in the model according to which the level of the debt is defined at the stage 3 and is endogenous to a prior estimation of the default probability of the borrower (variable determined at the stage 2).

[8] Analytically, we have: $(1+r_i)D_i = B_i = R^i(q_i, q_j, \widehat{z_i}) = \Phi_i(q_i(\widehat{z_i}, \widehat{z_j}), q_j(\widehat{z_i}, \widehat{z_j}), \widehat{z_i}) = \Psi_i(\widehat{z_i}, \widehat{z_j})$. This sequence of decisions brings into light a new determinant of the decision to take risks.

Secondly, the timing of the game is also coherent with the issuing of bond debts. Firms resorting to bond issues are quoted by specialized rating agencies which evaluate the issuer's quality on the basis of its bankruptcy risk. The bond's interest rate is conditional on the issuer's insolvency risk.

Thirdly, the literature concerning the relationship between banks and firms frequently emphasized that taking into account information in risk analysis increases estimation's precision of borrowers' quality. Regarding the bank-borrower relationship, Petersen (2004) puts forward a distinction between hard and soft information. Hard information is external and can be obtained via public information (balance sheet data) while soft information is internal via bank-borrower relationship. Hard information has the properties of verifiability and non manipulability and scoring is a hard information' treatment method for credit risk prediction. This method allows to implement risk adjusted pricing and leads to a decrease in credit rationing (Berger and al., 2002 ; Frame and al., 2001). Our model is coherent with credit risk management in banks through credit scoring where it is assumed that the available information is quantitative and verifiable[9]. The hard information structure refers to a situation in which the range of the states of nature must be evaluated and the distribution probability of each event must be computed (common knowledge in the model). The model is also consistent with bank's internal ratings used to predict default risk since in such a case the amount of the credit and the lending rate are dependent on the firm's rating.

2.3 Resolution of the game

We consider that the investment I_i with no residual value must be financed at least partially by borrowing D_i and shareholders provide $E_i = I_i - D_i$. In order to borrow D_i shareholders must promise to pay back $B_i = (1 + r_i)D_i$

At the production stage, the firm i maximizes the ex ante value of equity, denoted VNE^i and given by:

$$VNE^i = \int_{\widehat{z_i}}^{\overline{z}} \left[R^i - B_i\right] f(z)dz - (1 + r_i)E_i$$

In the above expression, the net value to shareholders is discounted at the interest rate of the loan (r_i).

The optimal level of production is given by (see appendix 1 for the demonstration):

$$Q + q_i = a + \alpha_i \tag{1}$$

Where : $\alpha_i = \frac{1}{4\overline{z}} \left(\overline{z} + \widehat{z_i}\right)^2$

By summing the best response functions for all the firms with positive debt, we obtain:

[9] This perception of risk comes from Knight (1921). He distinguishes risk and uncertainty and considers that a risky situation is measurable by using an objective probability.

$$n_1 Q + q_e = n_1 a + \sum_{i=1}^{n_1} \alpha_i$$

With : $q_e = \sum_{i=1}^{n_1} q_i$

If we now look at firm j which has no debt, her production decision (q_j) maximizes the expected value of operating profit $E(R^j)$. The first-best level of production q_j implies that:

$$Q + q_j = a \tag{2}$$

Summing the best response function for all the firms with no debt gives:

$$n_2 Q + q_{ne} = n_2 a$$

Where: $q_{ne} = \sum_{j=1}^{n_2} q_j$

Equations (1) et (2) give individual production levels of the two types i and j of firms given α_i (or implicitly given $\widehat{z_i}$).

At the first stage, we consider the choice of the optimal threshold state of nature $\widehat{z_i}$. Firms select the value of the bankrupt state $\widehat{z_i}$ with the objective of maximizing the sum of equity value and debt value[10]. The combined value of the firm's debt and equity corresponds to the expected cash flow (or total value of the firm) minus investment cost updated to the given rate d[11], which is the average rate of return set by the bank on her loan:

$$Y^i = \int_{-\overline{z}}^{\overline{z}} R^i(., z) f(z) dz - (1 + d) I_i$$

$$Y^i = \left[\int_{\widehat{z_i}}^{\overline{z}} R^i f(z) dz - \int_{\widehat{z_i}}^{\overline{z}} B_i f(z) dz \right] - (1 + d) E_i$$

$\Longrightarrow$ *Value of the firm to shareholders when equity capital is discounted at rate d*

$$+ \left[\int_{-\overline{z}}^{\widehat{z_i}} R^i(., z) f(z) dz + \int_{\widehat{z_i}}^{\overline{z}} B_i f(z) dz \right] - (1 + d) D_i$$

$\Longrightarrow$ *Value of the firm to the bank*

Differentiating this optimization program and the two aggregated best responses functions yields a system of equations that we solve to obtain product market equilibrium (proof see appendix 6.1)

[10] The bank is assumed to be perfectly informed on the firm's characteristics and on the shock on the demand. We dot not take into account agency problems in the lending relationship.

[11] The rate d is necessarily lower than r_i since at the rate r_i the bank's profit is always negative.

To simplify the exposition, we introduce the following notation: $\mu = nn_1 + n_2$.

The equilibrium aggregate output produced is:

$$Q = \frac{\mu}{1+\mu} a$$

The average price in the industry is given by:

$$E(p) = \frac{1}{1+\mu} a$$

The resulting production level for a firm with a positive debt can be compared with the production of an unleveraged firm:

$$q_i = \frac{n}{1+\mu} a$$

$$q_j = \frac{1}{1+\mu} a$$

It could be noticed that those equilibrium productions are independent of the demand volatility (scope of states of nature $\overline{z}$)

The resulting optimal level of default risk (θ_i) in the category of leveraged firms is:

$$\theta_i^2 = \frac{(n-1)a}{(1+\mu)\overline{z}}$$

By definition, the bankruptcy probability is given by[12]: $\theta_i = \frac{\overline{z}+\widehat{z}_i}{2\overline{z}}$

From the above expressions, it is straightforward to calculate firm's profit. Since the bank's profit is null at rate d, the value of the firm to equity holders is:

$$VNE^i = Y_i$$

If the firm has debt in its financial structure, the expected profit to shareholders can therefore be written as:

$$E(R^i) = \frac{n}{(1+\mu)^2} a^2$$

If the firm chooses not to be indebted, the expected return to shareholders is:

$$E(R^j) = \frac{1}{(1+\mu)^2} a^2$$

We obtain the interest rate on the loan as a function of the rate of return d of debt capital (proof can be found in the appendix 6.2):

$$\frac{1+r_i}{1+d} = \frac{n(q_i - \lambda_i)}{q_i - n\lambda_i}$$

where: $\lambda_i = \frac{1}{4\overline{z}} \left(\overline{z} - \widehat{z}_i\right)^2$.

[12] The condition $\theta_i \leq 1$ (or $\widehat{z}_i \leq \overline{z}$) implies the following constraint on $\overline{z}$: $\overline{z} \geq \alpha_i$.

3 Equilibrium properties of the model

For given values of n_1 et n_2, unleveraged firms follow a more conservative strategy, *i.e.* produce less than leveraged ones. This result is in line with Brander and Lewis (1986) model's which asserts that, in a Cournot duopoly, debt commits firm's shareholders to a more aggressive product market behavior (higher quantity choices). As shown by equation (1), the leveraged firm has an incentive to increase its output since a positive level of debt shifts the best response function outwards at the competitive stage of the game.

Provided that a the number of competitors (n) is given, when the proportion of leveraged firms in the industry increases, consumers benefit from a higher level of aggregate production but leveraged firms as well as unleveraged firms are affected with a lower individual expected profit. Especially if all the firms in the Cournot oligopoly have a positive debt, the individual operating profit is given by $E(R) = a^2n/(1 + n^2)^2$ while in the standard static Cournot game with no debt we obtain $E(R) = a^2/(1 + n^2)$ implying a higher expected profit. This result is commonly called the prisoner's dilemma : if firms could behave cooperatively, they would select only equity capital in their financial structure.

More generally, we consider the impact of the industry's composition, through n_1 et n_2, on the individual production decisions and on the bankruptcy probability for firms with debt obligation. The main properties of the subgame perfect equilibrium are summarized in the following proposition (proof: see Appendix 6.3).

Proposition 1 *When the number of leveraged firms, n_1, increases, the individual production of a leveraged firm decreases but the aggregate production of leveraged firms increases. Moreover, the industry output is higher despite the decrease of the aggregate production of unleveraged firms. Individual expected profits of firms decrease irrespective of their financial structure. When, $n \geq 3$, the bankruptcy probability is a decreasing function of n_1 .*

Proposition 2 *When the number of unleveraged firms, n_2, increases, the individual production of an unleveraged firm decreases but the aggregate production of unleveraged firms increases. Moreover, the industry output is higher despite the decrease of the aggregate production of leveraged firms. Individual expected profits of firms decrease irrespective of their financial structure. The bankruptcy probability is an increasing function of n_2 .*

4 An application to horizontal mergers

4.1 Horizontal mergers and market equilibria

An additional upstream stage to the game is introduced: initially, firms decide whether to merge or not. If a merger occurs between m firms, the resulting number of competitors in the industry is $(n - m - 1)$. The exact number of competitors in the industry results from the merger stage and all remaining firms engage in Cournot competition. All firms have the same constant marginal cost (for simplicity normalized at zero) and the decision to merge is not motivated by a decrease in the production cost. Then the game follows as in Section 2.

Using results from Section 3, we note that a merger in the group of leveraged firms or in the group of unleveraged firms generates, in both cases, anticompetitive effects (lower industry output). The comparative static properties of mergers are described in the following proposition:

Proposition 3 *:*

(i) A merger concerning firms with a positive debt exhibits higher anticompetitive effects,

(i.i) When the merger takes place between two leveraged firms, the individual production of a leveraged firm increases, the aggregate production of all leveraged firms decreases while the production of outsiders with no debt increases,

(i.i.i) When the merger takes place between two unleveraged firms, the individual production of an unleveraged firm increases, the aggregate production of all unleveraged firms decreases while the production of outsiders with debt increases.

The point *(i)* of the proposition results from the fact that Q is an increasing function of $\mu = (n_1 + n_2)n_1 + n_2$. When n_1 or n_2 falls in the same amount, the impact on μ is higher when the drop concerns n_1 rather than n_2 , *i.e.* when merging firms have debt in their financial structure. The two others points of the proposition arise from propositions stated in Section 3.

Accounting for the financial structure of firms engaging in a merger does not lead to reconsider the existence of anticompetitive effects but it affects their extent.

Concerning the impact of the merger on bankruptcy probability, we can state :

Proposition 4 *A merger between firms with a positive debt leads to an increase in the bankruptcy probability while a merger between firms with no debt involves a decrease in the bankruptcy probability. Finally, a merger between one leveraged firm and one or several unleveraged firms leads to a decrease in the bankruptcy probability of the merging entity.*

Horizontal mergers implying firms with positive debt in their financial structure increase the financial vulnerability of the new entity issued from the merger. Horizontal mergers involving unleveraged firms generate a positive externality for leveraged outsiders. In addition, the merger decreases the financial vulnerability when it includes a firm with no debt. This result provides an additional justification to the failing firm argument we find in most antitrust jurisdictions according to which one should facilitate horizontal mergers implying a firm in financial distress. Our model shows that such a merger constitutes an efficient alternative to the exit from the market through bankruptcy. Consequently, antitrust authorities should be accommodating towards horizontal mergers when firms involved by the mergers are characterized by different risks of failure.

4.2 Private incentives to merge

We consider private incentives to merge in an oligopoly setting where firms selecting no-debt financing compete with firms supporting debt obligations and consequently are endowed with a positive bankruptcy risk.

We first assume a symmetric oligopoly in order to compare incentives to merge in a context where all the competing firms either choose holding debt or either decide upon not to borrow money. In such a context, we examine to what extent the incentive to merge for anticompetitive considerations is motivated by the bankruptcy risk at the industry level.

In the case where all competing firms issue debt, individual levels of production and profit are given by (for simplification purpose, we use the notation: $E(R) = \Pi$):

$$\Pi(n) = \frac{na^2}{\left(1+n^2\right)^2}$$

In the benchmark case where firms select no debt in their financial structure, we have:

$$\Pi(n) = \frac{a^2}{\left(1+n\right)^2}$$

The private incentive to merge is given by the comparison between pre-merger and post-merger profits. If we assume a merged entity composed of m firms, the incentive to merge results from the condition[13] :

$$\Delta\Pi(n,m) = \Pi(n-m+1) - m\Pi(n) > 0$$

Salant et al. (1983) have shown that a profitable merger requires the merging parties' market share to be at least 80 percent. This result from the condition:

$$m\left(n-m+2\right)^2 < \left(1+n\right)^2$$

If we now turn to the case where firms select debt financing, the private incentive to merge is given by:

$$mn\left(1+\left(n-m+1\right)^2\right)^2 < \left(n-m+1\right)\left(1+n^2\right)^2$$

Consequently, the interaction between financial structure and output market equilibrium creates an additional motive for anticompetitive horizontal mergers (*cf. Fig.1*):

[13] Obviously, since the number of competitors is reduced, horizontal mergers for anticompetive motives benefit more to each outsider. This effect refers to the merger paradox implying that all profitable mergers cannot be reached since firms may prefer not to be involved in the merger (see models of endogeneous mergers, for instance Kamien and Zang, 1990; Krakel and Sliwka, 2006).

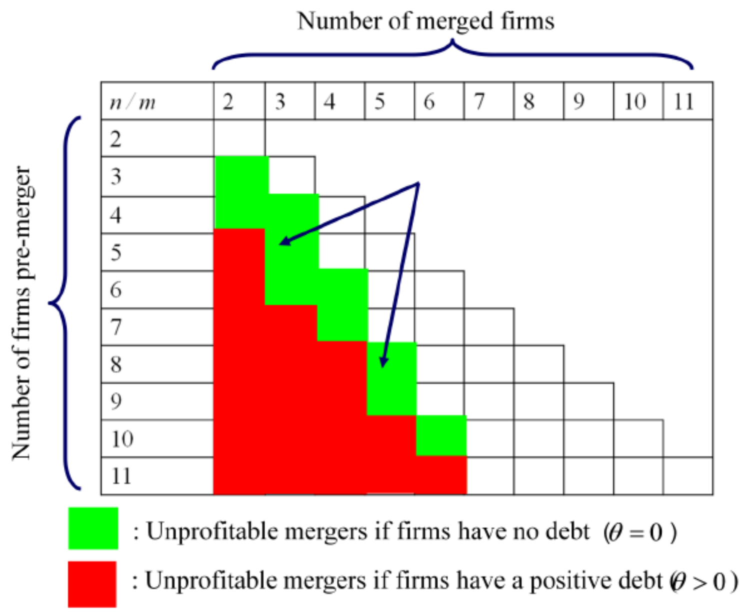

Fig.1 : Private incentive to merge under debt and no debt regimes

This figure shows that if firms are debt-financed, the minimum market share required for profitable mergers is reduced. This result comes from the fact that in a Cournot oligopoly debt intensifies the competition and lowers expected profits. For a given number of firms, the erosion of profits due to the high output and low price motivates horizontal concentration in order to increase the market power of firms and their profits. The benefit from anticompetitive effect of mergers is all the more important as the intensity of competition in the pre-merger context was intense.

We now consider private incentives to merge when firms issuing debt compete with equity financed firms. The objective here is to identify in what extend the incentive to merge is modified according to the financial structure of firms engaged in the merger.

The individual level of profit in the category of firms who are equity financed is given by:

$$\Pi^j(n_1, n_2) = \frac{1}{(1 + (n_1 + n_2)n_1 + n_2)^2} a^2$$

The private incentive to merge when m firms with no debt are engaged in the coalition is:

$$\Delta\Pi^j(n_1, n_2, m) = \Pi^j(n_1, n_2 - m + 1) - m\Pi^j(n_1, n_2) > 0$$

Or:

$$(1 + (n_1 + n_2)\, n_1 + n_2)^2 > m\,(1 + (n_1 + n_2 - m + 1)\, n_1 + n_2 - m + 1)^2$$

The individual level of profit of a firm i with debt financing is:

$$\Pi^i(n_1, n_2) = \frac{n_1 + n_2}{(1 + (n_1 + n_2)n_1 + n_2)^2} a^2$$

The private incentive to merge when m firms with positive debt are engaged in the coalition is:

$$\Delta\Pi^i(n_1, n_2, m) = \Pi^i(n_1 - m + 1, n_2) - m\Pi^i(n_1, n_2) > 0$$

Or:

$$\begin{array}{l}(n_1 - m + 1 + n_2)\,(1 + (n_1 + n_2)\, n_1 + n_2)^2 > \\ m\,(n_1 + n_2)\,(1 + (n_1 - m + 1 + n_2)\,(n_1 - m + 1) + n_2)^2\end{array}$$

We can observe that for $m = 2$: $\Delta\Pi^i(n_1, n_2, 2) = \Pi^i(n_1, n_2 - 1) - \Pi^i(n_1, n_2) - \Pi^j(n_1, n_2) > 0$.

Proposition 5 *A merger implying one firm of each category is always profitable independently of the number of firms in the industry*

The demonstration of this result is given in appendix 6.5

We now examine the private incentive to merge according to the financial structure of firms engaged in the coalition.

The condition on the minimum market share that the merging parties require in order to merge profitably in the category of firms with no debt is given by the ratio (m/n) so that[14] $\Delta\Pi^j(1, n_2, m) = 0$. Of course, the private incentive to merge for leveraged firms results from the sign of: $\Delta\Pi^i(n_1, 1, m)$

[14] In each case, we consider the most favorable context for a profitable merger, *ie* $n_1 = 1$ and $n_2 = 1$ respectively.

Mergers in the category of unleveraged firms ($n_1 = 1$)	
$n_2 = 2$	$(m/n)_{\min} = 67\%$
$n_2 = 3$	$(m/n)_{\min} = 75\%$
$n_2 = 4$	$(m/n)_{\min} = 80\%$
$n_2 = 5$	$(m/n)_{\min} = 83\%$
$n_2 = 6$	$(m/n)_{\min} = 71\%$

Table 1a: Minimum threshold of unleveraged participants for profitable mergers

Mergers in the category of leveraged firms ($n_2 = 1$)	
$n_1 = 2$	$(m/n)_{\min} = 67\%$
$n_1 = 3$	$(m/n)_{\min} = 50\%$
$n_1 = 4$	$(m/n)_{\min} = 60\%$
$n_1 = 5$	$(m/n)_{\min} = 50\%$
$n_1 = 6$	$(m/n)_{\min} = 57\%$

Table 1b: Minimum threshold of leveraged participants for profitable mergers

The ratio $(m/n)_{\min}$ is reduced compared to Salant, Switzer and Reynolds (1983). When outsiders have a positive debt, the minimum threshold of unleveraged participants for profitable mergers can be lower than 80%. In SSR model, when there is 3 or 4 competitors in the industry, mergers are never profitable unless they lead to monopolization of the market. Table 1b shows that when insiders have debt in their financial structure, the incentive to merge is higher compared to the case where merging parties have no debt. In a Cournot oligopoly where leveraged firms compete with unleveraged ones, the lowest percentage of the participants (m/n) for the merger to be profitable is always inferior when the insiders are debt-financed.

4.3 Welfare and implications for competition policy

Social welfare is given by the following expression:

$$W = (a - Q)Q + \frac{Q^2}{2} = \frac{a^2 - (a - Q)^2}{2}$$

Since $Q \leq a$, W is an increasing function of Q. Horizontal mergers results in a reduction in the number of firms and are never welfare improving since we assume no efficiency gains. We compare reductions in production (and consequently welfare losses) due to a merger involving two firms. We distinguish a merger with two leveraged insiders from a merger between a leveraged firm and an unleveraged ones.

$$a - Q(n_1, n_2) = \frac{a}{1 + \mu(n_1, n_2)}$$

With:

$$\mu(n_1 - 1, n_2) < \mu(n_1, n_2 - 1) < \mu(n_1, n_2)$$

Then:

$$a - Q(n_1 - 1, n_2) > a - Q(n_1, n_2 - 1) > a - Q(n_1, n_2)$$

Or:

$$W(n_1, n_2) > W(n_1, n_2 - 1) > W(n_1 - 1, n_2)$$

Social welfare losses are lower in the situation where the merger involves two firms with differentiated financial structures. The practical implication of this theoretical result is that antitrust authorities should not always prohibit mergers when merging firms face distinct bankruptcy risks. The failing firm argument constitutes an illustration of this view.

5 Concluding remarks

Financial structure of competitors engaged in a merger impacts on private incentives to merge, financial vulnerability of firms and social welfare. In a symmetric Cournot oligopoly, when firms have debt in their financial structure and consequently bear a positive bankruptcy risk, the minimum number of insiders for the merger to be profitable is lower than the corresponding number in the seminal model of Salant, Switzer and Reynolds (1983). In particular, a merger between a leveraged firm and an unleveraged firm is always profitable. An additional motive to horizontal mergers without efficiency gains appears: anticompetitive mergers are all the more profitable as shareholders, protected by limited liability, favors aggressive strategies towards competitors in the pre-merger game. The model also implies that one should expect that the probability to merge with competitors is higher in industries where the firms use debt in their financial structure. Concerning antitrust policy, our approach suggests that welfare losses in relation with anti-competitive horizontal mergers are lower when a firm which eventually may default is acquired by a firm with no bankruptcy risk rather than by a firm which bears a positive bankruptcy risk. Competition authorities, in assessing mergers involving failing firms, should take this outcome. Welfare losses in relation with a decrease in the number of competitors are higher when all insiders are debt-financed. As a consequence, such mergers create a wider discrepancy between private incentive and social desirability.

References

1. Berger, A.N., S. Frame, and N.H. Miller, 2002, *Credit scoring and the price and availability of small business credit*, Finance and Economics Discussion Paper Series 2002-26 Board of Governors of the Federal Reserve System.

2. Brander J.A. and T.R. Lewis , 1986, "Oligopoly and Financial Structure: the Limited Liability Effect", *American Economic Review,* 76, 956-970.

3. Deneckere R. and C. Davidson, 1985, "Incentives to Form Coalitions with Bertrand Competition", *RAND Journal of Economics*, 16(4), 473-486.

4. Fedele A. and M. Tognoni , 2005, *Failing Firm Defence with Entry Deterrence*, mimeo, Working Paper.

5. Fershtman C. and K.L. Judd, 1987, " Equilibrium Incentives in Oligopoly ", *American Economic Review*, 77(5), 927-940.

6. Frame, W.S., A. Srinivasan, and L.Woosley, 2001, The effect of credit scoring on small-business lending, Journal of Money, Credit and Banking 33, 813–825.

7. Franck B. and N. Le Pape , 2008, "The commitment value of the debt: a reappraisal", *International Journal of Industrial Organization*, 26, 607-615.

8. Gonzales-Maestre M. and J. Lopez-Cunat, 2001, " Delegation and Mergers in Oligopoly ", 19, *International Journal of Industrial Organization*, 1213-1244.

9. Kamien M.I. and I. Zang, 1990, "The limits of monopolization through acquisition", *Quarterly Journal of Economics*, 105(2), 465-499.

10. Kanatas G. and J. Qi, 2004, "Imperfect Competition, Debt, and Exit", *Financial Management*, 33, 29-49.

11. Knight F. H., 1921, *Risk, Uncertainty and Profit*, Boston, rééd.

12. Krakel M. and D. Sliwka, 2006, "Strategic Delegation and Mergers in Oligopolistics Contests", *Journal of Economics and Business*, 58, 119-136.

13. Levin, D., 1990, Horizontal Merger: the 50 percent benchmark, *American Economic Review*, 80, 1238-1245.

14. Maksimovic V., 1995, "Financial structure and product market competition" , in R.A. Jarrow, V. Maksimovic, and W.T. Zienba (éd.) : *Handbook in Operations Research and Management Science*, volume 9, North Holland, Elsevier, pp. 887-920

15. Mason R. et H. Weeds, 2003, *The Failing Firm Defence : Merger Policy and Entry*, mimeo, Working Paper.

16. Petersen, M.A., 2004, *Information : Hard and soft*, Mimeo Kellogg School of Management, Northwerstern University.

17. Salant S., S. Switzer and R.J. Reynolds, 1983, "Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium", *Quarterly Journal of Economics*, 98, 185–200.

18. Socorro, M. Pilar, 2007, "Mergers Under Uncertainty: The Effects of Debt Financing", *Manchester School*, 75 (5), 580-597.

19. Stiglitz J. and A. Weiss, 1981, Credit Rationing in Markets with Imperfect Information, American Economic Review, 71(3), 393-410.

20. Tirole J., 2007, *The Theory of Corporate Finance*, Princeton University Press, 640 pages.

21. Ziss S., 2001, "Horizontal Mergers and Delegation", *International Journal of Industrial Organization*, 19, 471-492

6 Appendix

6.1 Best response function

Firm i maximizes the ex ante value of equity (VNE^i):

$$VNE^i = \int_{\widehat{z_i}}^{\overline{z}} \left[R^i - B_i\right] f(z)dz - (1+r_i)E_i$$

$$VNE^i = \int_{\widehat{z_i}}^{\overline{z}} \left[R^i - B_i\right] f(z)dz - (1+r_i)I_i + B_i$$

In the above expression, the net value to shareholders is discounted at the interest rate of the loan (r_i).

The expected gross profit is denoted $\bar{R}^i$ and defined by:

$$\bar{R}^i = E(p)q_i$$

For analytical tractability we consider a linear demand, so we have:

$$R^i = \bar{R}^i + zq_i$$

$$R^i - B_i = R^i(q_i, q_j, z) - R^i(q_i, q_j, \widehat{z_i}) = (z - \widehat{z_i})q_i$$

$$VNE^i = \frac{1}{4\overline{z}} \left(\overline{z} - \widehat{z_i}\right)^2 q_i + \bar{R}^i + \widehat{z_i}q_i - (1+r_i)I_i$$

The shareholders decide on the optimal level of production to maximize the equity value, given the default risk on the loan: $\underset{q_i}{Max} VNE^i$

Maximization of the above expression yields first order condition[15]:

$$Rm_i = -\left[\frac{1}{4\overline{z}} \left(\overline{z} - \widehat{z_i}\right)^2 + \widehat{z_i}\right] = -\frac{1}{4\overline{z}} \left(\overline{z} + \widehat{z_i}\right)^2$$

Where : $Rm_i = \frac{\partial \bar{R}^i}{\partial q_i} = a - Q - q_i$.

Setting : $\alpha_i = \frac{1}{4\overline{z}} \left(\overline{z} + \widehat{z_i}\right)^2$, the optimality condition can be rewritten: $Rm_i = -\alpha_i$,

Or equivalently: $Q + q_i = a + \alpha_i$

[15] The investment is assumed to be exogeneously given and independent of the production.

6.2 Equilibrium productions

We differentiate optimality conditions of the stage 3 considering a marginal variation in α_i, $d\alpha_i$, others α_k being constant:

-for a leveraged firm, we have $Q + q_i = a + \alpha_i$ implying $dQ + dq_i = d\alpha_i$.

If we aggregate : $n_1 Q + q_e = n_1 a + \sum_{i=1}^{n_1} \alpha_i$ and $n_1 dQ + dq_e = d\alpha_i$,

Or: $dQ + dq_i = n_1 dQ + dq_e$

-for an unleveraged firm, we have $Q + q_j = a$,

If we aggregate : $n_2 Q + q_{ne} = n_2 a$ and $n_2 dQ + dq_{ne} = 0$

Or: $dq_{ne} = -n_2 dQ$.

We then differentiate the objective function at the stage 2:

$(a - Q)dq_i - q_i dQ = 0$,

$(a - Q)dq_i = q_i dQ$.

We consider the equation $dQ + dq_i = n_1 dQ + dq_e$, we have $dQ = dq_e + dq_{ne}$, and therefore

$dq_i = (n_1 - 1)dQ + dq_e = (n_1 - 1)dQ + dQ - dq_{ne}$

$dq_i = n_1 dQ + n_2 dQ = (n_1 + n_2)dQ$

Consequently

$q_i dQ = (a - Q)(n_1 + n_2)dQ$

$q_i = (a - Q)(n_1 + n_2)$

Or : $n_1 q_i = n_1(a - Q)(n_1 + n_2) = q_e$

Since: $q_{ne} = n_2(a - Q)$,

By adding q_e and q_{ne} we obtain:

$Q = n_1(a - Q)(n_1 + n_2) + n_2(a - Q)$,

Therefore, we get the total industry output:

$$Q = \frac{\mu}{1 + \mu} a$$

With $\mu = n_1(n_1 + n_2) + n_2 = n_1 n + n_2$.

6.3 Loan interest rate

The bank's profit, discounted at rate d, is :

$\Pi_b = \int_{-\overline{z}}^{\widehat{z}_i} R^i f(z)dz + \int_{\widehat{z}_i}^{\overline{z}} B_i f(z)dz - (1 + d)D_i = 0$

or

$\Pi_b = \int_{-\overline{z}}^{\widehat{z}_i} R^i f(z)dz + \int_{\widehat{z}_i}^{\overline{z}} (\overline{R^i} + \widehat{z}_i q_i) f(z)dz - (1 + d)D_i = 0$

which, after some manipulations gives

$$\overline{R^i} - \lambda_i q_i = (1 + d)D_i = \rho_i B_i$$

where $\lambda_i = \frac{(\overline{z} - \widehat{z}_i)^2}{4\overline{z}}$, $\rho_i = \frac{1+d}{1+r_i}$.

As a consequence,

$$\rho_i = \frac{\overline{R^i} - \lambda_i q_i}{\overline{R^i} + \widehat{z}_i q_i}$$

Since

$$\alpha_i = \frac{(\overline{z} + \widehat{z}_i)^2}{4\overline{z}} = \lambda_i + \widehat{z}_i$$

one expression for ρ_i is

$$\rho_i = \frac{E(p) - \lambda_i}{E(p) + \alpha_i - \lambda_i}$$

At equilibrium, $E(p) = \frac{1}{1+\mu} a = \frac{q_i}{n}$, and $E(p) + \alpha_i = q_i$,
so that finally,

$$\frac{1}{\rho_i} = \frac{1 + r_i}{1 + d} = \frac{n(q_i - \lambda_i)}{q_i - n\lambda_i}$$

6.4 Incidence of n_1 and n_2 upon productions and the probability of failure

6.4.1 Proof of Proposition 1

Given the expression $Q = \frac{\mu}{1+\mu} a$, it is straightforward that Q is an increasing function of μ and consequently of n_1. Considering equation (2) it results that when n_1 increases, q_j and q_{ne} decreases, and as a consequence q_e increases.

If we now turn to q_i, we have: $q_i = \frac{n}{1+\mu} a$

It is easier to consider the expression a/q_i

$$\frac{a}{q_i} = \frac{1 + \mu}{N} = \frac{1 + n_2}{n_1 + n_2} + n_1 \tag{A1}$$

It can be easily verify that $d(\frac{a}{q_i})/dn_1 = 1 - \frac{1+n_2}{(n_1+n_2)^2} \geq 0$

We have that q_i is decreasing function of n_1.

The probability of failure is increasing with α_i, and note that:

$$\alpha_i = q - (a - Q) = \frac{n - 1}{1 + \mu} a$$

We then have:

$$\beta_i = \frac{1}{\alpha_i} = \frac{nn_1 + n_2 + 1}{n - 1} = n_1 + \frac{n + 1}{n - 1} \tag{A2}$$

By computing the derivative with respect to n_1, we obtain:

$$\frac{d\beta_i}{dn_1} = 1 - \frac{2}{(n - 1)^2}$$

We have: $\frac{d\beta_i}{dn_1} > 0$ if $n \geq 3$.

Under this condition on the number of firms, α_i (or equivalently the probability of failure) decreases with n_1.

6.4.2 Proof of Proposition 2

Recall that Q is increasing with μ. Since, μ increases with n_2, it is obvious that when n_2 increases, Q increases.

If we now consider q_i, equation (A1) implies that the expression a/q_i increases with n_2.

As a consequence, when n_2 increases, q_i decreases and also q_e. It then results that q_{ne} increases with n_2. From the equation (2), one can see that when n_2 increases q_j decreases.

Equation (A2) exhibits that β_i decreases with n and so with n_2. Consequently, α_i increases with the number n_2.

6.5 Profitable two-firms mergers

We have to show that:

$$\Delta\Pi^i(n_1, n_2 - 1) = \Pi^i(n_1, n_2 - 1) - \Pi^i(n_1, n_2) - \Pi^j(n_1, n_2) > 0$$

With:

$$\Pi^i(n_1, n_2) = \frac{n_1 + n_2}{(1 + (n_1 + n_2)n_1 + n_2)^2} a^2$$

And:

$$\Pi^j(n_1, n_2) = \frac{1}{(1 + (n_1 + n_2)n_1 + n_2)^2} a^2$$

The inequality is then :

$$\frac{n_1 + n_2 - 1}{(1 + (n_1 + n_2 - 1)n_1 + n_2 - 1)^2} a^2 > \frac{n_1 + n_2 + 1}{(1 + (n_1 + n_2)n_1 + n_2)^2} a^2$$

Since: $\mu = n_1(n_1 + n_2) + n_2 = n_1 n + n_2$, we have:

$$\frac{n_1 + n_2 - 1}{(1 + \mu - n_1 - 1)^2} > \frac{n_1 + n_2 + 1}{(1 + \mu)^2}$$

$$\left(\frac{1 + \mu}{1 + \mu - n_1 - 1}\right)^2 > \frac{n_1 + n_2 + 1}{n_1 + n_2 - 1}$$

$$\left(1 - \frac{n_1 + 1}{1 + \mu}\right)^2 < \frac{N - 1}{N + 1}$$

We introduce the following notation:

$$x = \frac{n_1 + 1}{1 + \mu} = \frac{n_1 + 1}{1 + N + (N - 1)n_1}$$

For a given value of N, x is an increasing function of n_1 :

$$\frac{dx}{dn_1} = \frac{2}{(1 + N + (N - 1)n_1)^2} > 0$$

Consequently, the minimum value for x is obtained when $n_1 = 1$. This value is: $x_{\min} = 1/N$

$$(1 - x_{\min})^2 = \left(\frac{N - 1}{N}\right)^2$$

Obviously: $\left(\frac{N-1}{N}\right)^2 < \frac{N-1}{N+1}$
Then:

$$\left(1 - \frac{n_1 + 1}{1 + \mu}\right)^2 < \frac{N - 1}{N + 1}$$

TEPP Working Papers

10-1. Are young French jobseekers of ethnic immigrant origin discriminated against? A controlled experiment in the Paris area
Emmanuel Duguet, Noam Leandri, Yannick L'Horty, Pascale Petit

10-2. Couple's Work Hours, Satisfaction and reconciling Work and family Life
Nathalie Georges, Dominique Méda, Danièle Trancart

10-3. Housing ownership, social housing and unemployment: an econometric analysis of the Paris area
Emmanuel Duguet, Yannick L'Horty, Florent Sari

10-4. Do Public Subsidies Have an Impact on New Firm Survival? An Empirical Study with French Data
Lionel Désiage, Richard Duhautois, Dominique Redor

10-5. The effect of social security payroll tax reductions on employment and wages: an evaluation of the 2003 French reform
Matthieu Bunel, Fabrice Gilles, Yannick L'Horty

10-6. What are Entrepreneurs' Objectives When Starting a New Business?
Lionel Désiage

10-7. Health and Early Retirement: Evidence from French Data for individuals
Thomas Barnay, Karine Briard

10-8. Ageing, chronic conditions and the evolution of future drugs expenditures
Thomas Barnay, Sophie Thiébaut, Bruno Ventelou

10-9. Entrepreneurial motives and performance: Why might better educated entrepreneurs be less successful?
Arnab Bhattacharjee, Jean Bonnet, Nicolas Le Pape, Régis Renault

10-10. Returns to firm-provided training in France: Evidence on mobility and wages
Arnaud Chéron, Bénédicte Rouland, François-Charles Wolff

10-11. Taxation of early retirement windows and delaying retirement: the French experience
Pierre-Jean Messe

10-12. Pre Entry Motives into Entrepreneurship and Post Entry Entrepreneurial Orientation
Jean Bonnet, Nicolas Le Pape

10-13. Hiring Practices, Employment Protection and Temporary Jobs
Anne Bucher

10-14. Young-in Old-out: a new evaluation
Michela Bia, Pierre-Jean Messe, Roberto Leombruni

10-15. On the impact of the TFP growth on the employment rate: does training on-the-job matter?
Eva Moreno-Galbis

10-16. The dynamics of youth labor market integration
Anne Bucher

10-17. Fostering the potential endogenous development of European regions: a spatial dynamic panel data analysis of the Cohesion Policy on regional convergence over the period 1980-2005
Salima Bouayad-Agha, Nadine Turpin, Lionel Védrine

10-18. Cost-saving or Cost-enhancing Mergers: the Impact of the Distribution of Roles in Oligopoly
Nicolas Le Pape, Kai Zhao

10-19. Bankruptcy Risk, Product Market Competition and Horizontal Mergers
Bernard Franck, Nicolas Le Pape

10-20. Endogenous Job Destructions and the Distribution of Wages
Arnaud Chéron, Bénédicte Rouland

10-21. Employment Protection Legislation and Adverse Selection at the Labor Market Entry
Anne Bucher, Sébastien Ménard

The TEPP Institute

The CNRS **Institute for Labor Studies and Public Policies** (the TEPP Institute, FR n°3126 CNRS) gathers together research centres specializing in economics and sociology:

- the **Centre d'Etudes de l'Emploi** (Centre for Employment Studies), **CEE**, Public Administrative Organization, under the leadership of the Ministers of Work and Research
- l'**Equipe de Recherche sur les Marchés, l'Emploi et la Simulation** (Research Team on Markets, Employment and Simulation), **ERMES**, University of Paris II Panthéon-Assas
- the **Centre d'Etudes des Politiques Economiques de l'université d'Evry** (Research Centre focused on the analysis of economic policy and its foundations and implications), **EPEE**, University of Evry Val d'Essonne
- the **Centre Pierre Naville** (Research on Work and Urban Policies), **CPN**, University of Evry Val d'Essonne
- l'**Equipe de Recherche sur l'Utilisation des Données Temporelles en Economie** (Research Team on Use of Time Data in Economics), **ERUDITE**, University of Paris-Est Créteil and University of Paris-Est Marne-la-Vallée
- the **Groupe d'Analyse des Itinéraires et des Niveaux Salariaux** (The Group on Analysis of Wage Levels and Trajectories), **GAINS**, University of the Maine

The TEPP Institute brings together 115 researchers and research professors, 140 PhD students and 40 associate researchers who study changes in work and employment in relation to the choices made by firms and analyse public policies using new evaluation methods.