ENDOGENOUS WAGE RIGIDITIES, HUMAN CAPITAL ACCUMULATION AND GROWTH

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Endogenous wage rigidities, human capital accumulation and growth

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Abstract

This paper explains the dynamics of wage rigidity and youth employment as the result of economic and political interactions of generations of workers. Over the past decades some OECD countries simultaneously experienced a rise in labor productivity and a rise in unemployment. In this context the political economy of wage rigidity is intriguing as (1) those with the political power are savers and those without accumulate human capital and (2) dynamic effects are crucial as workers may also vote in order to alter the future preferences for wage rigidities. We show the existence of three types of equilibrium dynamics: one converging to full employment; an other characterized by an unemployment trap and also multiple equilibrium dynamics where, depending on their initial level of human capital, countries with the same fundamental may converge to steady state with sharp differences in employment, productivity and income per capita.

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1 Introduction

Since the early 1970s, the contrast in the employment experiences of different OECD countries is striking. Indeed, while some E.U. countries faced a continuous rise of unemployment and its persistence at high levels, other OECD countries (most notably the U.S. and the U.K.) experienced a less significant rise in unemployment that came to an end at a certain point in time. Here is a tale of a fortune reversal: in the early 1970s, OECD standardized unemployment rates were between 2% and 3% for most European countries. By the end of the nineties unemployment rate had risen in all of these countries, averaging 10.7% in the European Union (see OECD, 2000). The experience of the U.S. somewhat contrasts with that of others OECD countries. In the early 1970s unemployment rate in US was slightly higher than most of the other OECD countries. By the end of the 1990s it is roughly half that of the other OECD countries. Moreover, stick in to the comparison of the E.U. and the U.S. and focusing on demographic unemployment pattern, Figure 1 shows that the young have paid a disproportionately high tribute to the rising European unemployment. Instead the demographic unemployment pattern in the U.S. has remained remarkably stable over the period.

![Unemployment Rate by Age Group](image)

Figure 1: Time profile of unemployment rate by age group (Source: OECD Labor Force Stat)

These demographic unemployment patterns divergence have gone hand in hand with a divergence in accumulation patterns. Indeed, starting in the late 1970s, hourly productivity in those

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1 Over this period European labor markets have been diagnosed by many as *sclerotic*, due to their lack of labor reallocation. Overall there is much less inflow into and outflow from unemployment in Europe than in the US (Elsby et al. 2013; Petrongolo & Pissarides 2008).
European countries has increased and constantly remained above that of the U.S.\footnote{Starting in the mid 1970s, an increasing trade-off between productivity and employment has been documented \cite{Beaudry2005}. But, the European lead in hourly labor productivity end up starting from the mid 1990’s in favor of US, except for some Northern European countries \cite{CESifo2002}.} However overall this productivity advantage has been eroded by fewer working hours and lower employment rate. Consequently this situation does not materialize in a higher income per capita and growth performance. Figures of Table (1) do not only emphasize differences in overall growth rates but also differences in growth strategies. Through the 1980s and 1990s European growth has been physical capital intensive, while the US’s growth has been human capital intensive.

<table>
<thead>
<tr>
<th>Country</th>
<th>GDP</th>
<th>Employment</th>
<th>Labor productivity</th>
<th>Capital deepening</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>3.2</td>
<td>2</td>
<td>1.2</td>
<td>0.9</td>
</tr>
<tr>
<td>France</td>
<td>2</td>
<td>-0.1</td>
<td>2.1</td>
<td>2.3</td>
</tr>
<tr>
<td>Spain</td>
<td>2.1</td>
<td>0</td>
<td>2.1</td>
<td>3.1</td>
</tr>
<tr>
<td>Italy</td>
<td>2.3</td>
<td>0.1</td>
<td>2.2</td>
<td>4.3</td>
</tr>
</tbody>
</table>

This sharp and persistent differences suggest structural factors as a possible explanation. A natural way to go along economies structural features in relation to their labor market outcomes, is to consider their labor market institutions. Hence we echo \cite{Blanchard2006} stating that:

"...a clear shift in focus took place, both among policy makers and among researchers, for two reasons. First, continuing high unemployment in the major continental countries made the earlier explanations, based on adverse shocks and persistence, increasingly implausible: Could shocks in the 1970s and the 1980s still have such strong effects in the 1990s and 2000s? And second, given the continued large commonality of shocks, the differences in unemployment rates across countries pointed to differences in institutions as central to any explanation of unemployment..."

Accordingly this paper proposes a common explanation for the rise and fall of unemployment with its connection to labor productivity growth in terms of endogenous rise and fall of wage rigidities that disproportionately hurt young workers. In our paper the medium run trade-off between unemployment and productivity form the basis for a redistributive conflict between different generations of workers. To encompass the dynamic feature of the trade-off we develop a three period overlapping generations neoclassical growth model with heterogeneous agents and with an endogenously determined level of wage rigidity. Agents work during the two first periods of their life, being outsiders while young and insiders as they get older. They retire in the third period. During the two first periods they successively accumulate human capital and financial assets. Being less exposed to unemployment, prime-aged workers have a stake on wage rigidities as it involves job rationing which raises their current labor income. However, once they retire they are capitalists (or firms owners), thus they have a stake on labor market flexibility since it increases their capital income. Higher wage rigidities raises youth unemployment and decreases their ability to invest in human capital. Therefore unemployment affects negatively human capital accumulation and returns of saving due to the complementarity of human and physical capital. In our model, rational and forward looking insiders understand, and take into account this detrimental effect of youth unemployment on earnings over the life cycle. From a
policy point of view current insiders also take into account that wages in the future will be set by next generation of insiders. This choice will depend on the future state of the economy, which is determined by the current collective and private choices. Thus, current and future political choices are linked via the impact of wage rigidities on the relative accumulation of human and physical capital, which in turn determine future preferences.

In this context, higher wage claims act as a tax which redistributes income away from the young (outsiders) and capitalists (retiree) toward savers (insiders) and boosts investments. Hence, an economy whose insiders adopt a high wage policy has a growth based on physical capital accumulation. Instead, an economy relying on lower wage policy has a growth based on human capital accumulation. Indeed, higher rate of young employment relaxes credit constraint which fosters human capital investment and next period stock of human capital. Whether one growth strategy or the other emerges as a political equilibrium depends on the human capital loss induced by unemployment. When the loss is high, the economy is more likely to rely on a human capital driven growth converging to full employment. Conversely, for smaller losses, growth is mainly driven by physical capital accumulation, with a high productivity of labor per efficiency unit and the existence of an unemployment trap for the youth. However intergenerational strategic political complementarities can also generate multiple steady state equilibria in which economies with the same fundamentals may end-up with very different labor market outcomes. The strategic political complementarity choices between generations of insiders is the consequence of the substitutability of insiders and outsiders labor on one hand and of capital-labor complementarity on the other hand, coupled with the forward looking behavior of insiders. Current wages increases set the ground for further increases in the future. Indeed, since next generations of insiders start with lower average human capital and more physical capital to work with, their period labor demand curve is less elastic, which increases their monopoly power and there incentives for supporting greater wage rigidities. Unlike myopic insiders, forward looking and rational insiders fully integrate this complementarity of behavior in their choices. Therefore, strategic complementarities of behavior is the specific feature of our model that generate the multiplicity of equilibria.

The model predictions are broadly consistent with empirical studies comparing hourly productivity, employment rates and aggregate output in the U.S. and a number of European countries. Notably, Blanchard (1997) and Daveri et al. (2000) documented the rise in capital share in Europe for the period 1980-1995, which does not have its counterpart in the U.S.. Blanchard (1997) claims that the change stems from labor demand shifts due to capital biased technical progress and the change in the distribution of rents from workers to firms. Rather, in our model this is due to the different cross country saving patterns that are mediated through the endogenous adoption of labor market institutions that redistribute toward the labor force segment whose propensity to save is the highest. A related paper by Gordon (1995) documents that the positive trade-off between productivity and unemployment is only temporary and that an adjustment mechanism is at work involving capital accumulation; where after successive periods of productivity slow down, productivity starts recovering and goes hand in hand with employment. In Appendix (B), an extension of the model to endogenous growth is presented which allows to account for this temporary employment productivity trade-off. While initially a lower

\[3\] This is due to the fact that prime aged human capital is a quasi fixed input.

\[4\] See also Beaudry et al. (2005) for evidence on this trade-off.
unemployment rises productivity, later on, as the human capital stock increases, physical capital accumulates at higher rates than human capital and productivity starts to be positively associated with the employment rate and human capital accumulation. Neither Blanchard (1997) nor Gordon (1995) consider this joint evolution of human and physical capital, while this is the key mechanism explaining the persistence of unemployment and institutions in our model.

The model posits that labor market institutions are designed by prime-aged workers. Following Saint-Paul (2002) political insiders models, prime-aged workers are less exposed to unemployment and are the political insiders, while youths are considered as the outsiders. The political insider model, because it avoids to deal with voting aggregation issues, is a convenient framework for introducing the intergenerational linkages between workers holding different resources and facing different investment choices at different period of their life and to study conditions for the rise or the decline of wage rigidities. In our model, the wage rate is endogenously chosen through repeated voting by rational and forward looking agents. Current policy affects future policies through its impact on private investment choices in human and financial assets. The model builds on previous work on dynamic political choices in macroeconomics and focus on Markov perfect equilibria, following the seminal contribution of Rios-Rull & Krusell (1999). This literature emphasizes the strategic interaction induced by repeated voting. This literature mostly rely on numerical methods for solving the equilibrium dynamics. Instead, we build on Hassler et al. (2003) and provide analytical characterization of the set of equilibria.

Still few papers investigate the long run effect of labor market institutions. Most of research has concentrated on their effect on labor market fluctuations [Hopenhayn & Rogerson, 1993]. However, in so far as those institutions affect youths labor market prospects and lifetime earnings they should also determine their investment choices in human capital, their future labor market prospect and in-fine their political preferences for those institutions. In this respect, labor market institutions create political and economic intergenerational linkages which drives the effects of those institutions well beyond the short run. Our contribution is therefore clearly related to the literature studying the interaction of labor market institutions and accumulable factors of productions that are essential to growth such as physical and human capital [Cahuc & Michel 1996; Caballero & Hammour 1998; Saint-Paul 1996, 2002; Brügemann 2012; Janiak & Wasmer 2014]. In particular, Saint-Paul (2002) and Brügemann (2012) analyse the endogeneity of labor market institutions and its relation to growth and productivity. A noteworthy conclusion of these papers that relates to ours is that initial support for labor rigidity may generate its own future constituency. In our model workers poorly endowed with human capital favor wage policies that generate subsequent support for more rigidities. Our paper shares the general concern of Blanchard (1997), Caballero & Hammour (1998) and more recently Janiak & Wasmer (2014) by making clear that institutional factors are as relevant for the long run (growth) as they are for the short run (unemployment).

The rest of the paper is organized as follows. The next section presents the general framework of the model. The third section derives the equilibrium economic and policy rules and contains...
the main results. The last section summarizes the results and discusses possible extensions. An appendix contains the proofs and an extension of the model to endogenous growth.

2 Wage rigidities in a neoclassical growth model with heterogeneous generations of workers

The economy operates in a perfectly competitive environment and economic activity extends over infinite discrete time. We consider a three period overlapping generations. Depending on their age and labor market status we distinguish: young workers, prime-aged workers and retiree. In each period, firms hire labor from young and prime-aged workers and capital from retirees to produce a unique final good. The population is stationary and each generation has a measure equal to one.

2.1 Firms

A large number of firms produce a unique final good. The production technology is described by a CRS production function $F$ with physical capital $K$ and human capital (efficiency units) $H$ as inputs:

$$Y_t = F(K_t, H_t) = A(K_t)^{\alpha} H_t^{1-\alpha}$$  \hfill (1)

with $H_t = L^o(\tilde{l}_t) + L^i(E_t) = H(\tilde{l}_t, E_t)$

The production of efficient labor $H$ is additive with prime-aged insiders and young outsiders human capital as arguments, $L^i(E_t)$ and $L^o(\tilde{l}_t)$. Each worker has an inelastic labor supply. Each young worker is initially endowed with one unit of efficient labor. We denote $\tilde{l}_t$ the young unemployment rate. Hence, $L^o(\tilde{l}_t) = \tilde{l}_t$ is the quantity of efficient units of labor provided by the young. The young can improve their efficiency at work, which will materialize next period by investing in human capital. Thus, depending on his past human capital investment a prime-age worker is either skilled or unskilled. Skilled and unskilled workers provide respectively $\eta_e$ and $\eta_u$ efficiency units, with $\eta_e > \eta_u$. Hence if we note $E_t$ the mass of skilled insiders at $t$, and since each generation is of measure one, insiders human capital supply is $L^i(E_t) = \eta_e E_t + (1 - E_t) \eta_u$. Aggregate labor supply measured in efficiency unit is:

$H_t = \tilde{l}_t + \eta_u + E_t \Delta \eta$

where $\Delta \eta = \eta_e - \eta_u > 0$.

With perfect competition, wage rate is equal to the marginal labor productivity and the interest rate is equal to the marginal productivity of capital net of depreciation rate. It is assumed that capital is fully depreciated in one period of time.

$$w_t = F_H(K_t, H_t)$$  \hfill (2)

and $R_t = F_K(K_t, H_t)$

2.2 Individuals

Preferences

Each individual lives for three periods and earns labor income in the two first periods of his life. At birth, knowing their idiosyncratic investment costs, agents decide whether to invest in
human capital or not. The second period agents become insiders with certainty independently of their previous labor market experience. During this second period, insiders vote over a wage rate for this period, consume and save for their old age. Entering the third period, agents leave the labor market, loose their insider status and its associated right to vote and consume the proceeds of their savings. The preferences of the generation born in a generic period \( t - 1 \) are represented by a logarithmic utility function:

\[
U(c^0_{t-1}, c^1_t, c^r_{t+1}) = \ln(c^0_{t-1}) + \beta \ln(c^1_t) + \beta^2 \ln(c^r_{t+1}),
\]

where \( \beta \) is the discount factor and the subscript refers to the timing of consumption and the upper-scripts stand respectively for young (outsider), prime-aged (insider) and old (retiree).

To concentrate on insiders saving behavior we further assume that youth consume their whole income. Each young worker is endowed with one unit of time. We assume that unemployment spells are uniformly distributed among young workers. Hence if \( \bar{l}_t \) is the employment rate, firms hire a fraction \( \bar{l}_t \) of each young individual unit of time endowment which is worth one efficiency unit. Hence the young first period income is simply \( \bar{l}_t w_t \).

The insider (or prime-aged worker) income, \( y^i_t \), is divided between second period consumption and saving for his retirement consumption. An insider solves the following program:

\[
\begin{align*}
\max_{s_t} & \quad \ln c^i_t + \beta \ln c^r_{t+1} \\
\text{s.t.} & \quad s_t + c^i_t = y^i_t \\
& \quad c^r_{t+1} = s_t R_{t+1} \\
& \quad c^i_t \geq 0, c^r_{t+1} \geq 0
\end{align*}
\]

With logarithmic preferences, the saving decisions of insiders are independent of their saving’s returns and each insiders saves a fraction \( \frac{\beta}{1+\beta} \) of his labor income for his retirement. The insiders indirect utility function writes:

\[
V(y^i_t; R_{t+1}) = (1 + \beta) \ln y^i_t + \beta \ln R_{t+1} + c
\]

where \( c = \ln(1 + \beta)^{(1+\beta)} \beta^\beta \) is a constant.

Youths educational choices

Knowing their educational cost \( \sigma_i \), agents decide whether to invest in human capital or not. I assume that \( \sigma_i \) is iid within and between generations, in particular it is not linked to any parental characteristic, \( \sigma_i \) follows a P.D.F. \( f \). The total cost of education at time \( t \) is:

\[
\sigma_i W_t
\]

So that is a particular definition of “insiders”, which is not the same as in the dynamic insider-outsider theory.

The log-linear utility function is chosen for computational convenience, the crucial assumption being that the utility function is homothetic in the second and third period consumptions. With this hypothesis, a constant fraction of the second period income is saved for the third period consumption.

Rather than assuming job rationning, we thus implicitly assume that the is rationning in the quantity of labor provided at each job due to the minimum wage. In this way, we do not need to introduce an insurance or unemployment benefit scheme, whose absence would lead to zero consumption.
This cost is equal to the average wage $W_t$ times an individual specific cost $\sigma_t$, and is paid by the young out of his first period income. Education or training raises second period productivity as $\eta_e > \eta_u$. Labor supply is assumed to be inelastic and in $t+1$ an educated (resp. uneducated) agent earns $\eta_e W_{t+1}$ (resp. $\eta_u W_{t+1}$). A young born at $t$ choosing to acquire education has the indirect utility function:

$$V^1_e = \ln \left( \tilde{l}_t W_t - \sigma_t W_t \right) + \beta V(\eta_e W_{t+1}; R_{t+2})$$

while the unskilled indirect utility is:

$$V^1_u = \ln \left( \tilde{l}_t W_t \right) + \beta V(\eta_u W_{t+1}; R_{t+2})$$

Acquiring human capital is costly but it increases second period productivity at a higher rate. A young will invest if and only if the utility derived from doing so is higher or equal to the utility derived from not investing,

$$V^1_e \geq V^1_u,$$

\[ \Leftrightarrow \]

$$\log \left( 1 - \frac{\sigma_t}{\tilde{l}_t} \right) + \beta (1 + \beta) \log \frac{\eta_e}{\eta_u} > 0$$

This condition determines a critical cost function, $\sigma^*_t(\tilde{l}_t)$, such that only those individuals with an education cost $\sigma_t$ below this threshold will choose to invest in human capital:

$$\sigma^*_t(\tilde{l}_t) = \tilde{l}_t \left[ 1 - \left( \frac{\eta_u}{\eta_e} \right)^{\beta(1+\beta)} \right]$$

(5)

$\sigma^*_t(\tilde{l}_t)$ increases with the employment rate $\tilde{l}_t$. This is due to the fact that with credit constraints on human capital investments, the marginal cost of education decreases when youths employment opportunities rise. Better employment opportunities exert a positive wealth effect that lowers the utility cost of education. A higher wage gap serves as an incentive for investing since $\frac{\partial \sigma^*_t}{\partial \tilde{l}_t} > 0$. Those who attend school while young are educated insiders next period, their mass is $E_{t+1} = \int_0^{\sigma^*_t(\tilde{l}_t)} f(\sigma_i) d\sigma_i$. I assume that $\sigma_i$ follows a uniform distribution over the domain $[0, \tilde{\sigma}]$. The supply of educated at $t+1$ is then:

$$E_{t+1} = E(\tilde{l}_t) = \frac{\tilde{l}_t \left[ 1 - \left( \frac{\eta_u}{\eta_e} \right)^{\beta(1+\beta)} \right]}{\tilde{\sigma}} = x \tilde{l}_t$$

(6)

we refer to the function $E(.)$ as the human capital investment function. The investment cost is indexed by $\tilde{\sigma}$, a higher $\tilde{\sigma}$ means that it is more costly to invest and consequently human capital accumulation depend less on employment opportunities. The next period stock of skilled insiders $E_{t+1}$, rises with the current youth employment rate, $\tilde{l}_t$.  

---

Footnote 9: With imperfect markets, future labor income cannot serve as collateral as argued by Ljungvist (1993). I have not specifically introduced an educational sector. One may assume that skilled insiders are input in the education production sector which pay a competitive wage. Then, $\sigma_i$ is the time a youngster has to buy from an insider to be educated. In this case one has to take out from the insiders labor supply to firms the time spent teaching the youths and add a financing scheme. To simplify, we just assume an exogenous consumption cost of acquiring general education which should be paid by the end of the first period due to capital market imperfections.
Physical capital supply

From the assumption that young outsiders have no access to the capital market, physical capital at period \( t \), \( K_t \), is equal to the savings of period \( t - 1 \) insiders \( S_{t-1} \), i.e.,

\[
S_{t-1} = K_t \forall t
\]  

and since the unique savers are the prime-aged workers, the following market clearing equality is verified:

\[
\frac{\beta}{1+\beta} L^i(E_t)W_{t-1} = K_t
\]

from which we derive the dynamics of capital stock:

\[
K_{t+1} = \frac{\beta}{1+\beta} L^i(E_t)A(1-\alpha) \left( \frac{K_t}{H(l_t;E_t)} \right)^\alpha,
\]

the capital stock dynamics depend on the evolution of \( E_t \) and of the youth employment rate.

3 The political-economic equilibrium

3.1 The insiders objective function

We now specify the policy choice. The state of the economy at \( t \) is summarized by a stock of capital \( K_t \), and the insiders stock of human capital \( E_t \). With the assumption that insider status prevents prime-aged workers from being unemployed and with the assumption of substitutability between both types of labor, the choice of minimum wage is equivalent to a choice over youths employment rate \( \tilde{l}_t \). Despite their income heterogeneity, prime-aged workers share the same objective regarding the minimum wage policy. This is due to the fact that the insider status applies to all prime age workers irrespective of their education type and that with logarithmic preferences the propensity to save is independent of income. Hence, we can refer to a representative insider or a union seeking a minimum wage policy that maximizes the insiders lifetime utility.

Lemma 1 The representative insider objective function is:

\[
V(\tilde{l}_t, \tilde{l}_{t+1}; E_t, E_{t+1}) = -S \ln H_t(\tilde{l}_t; E_t) + \ln H_{t+1}(E(\tilde{l}_t), \tilde{l}_{t+1})
\]

with \( S = \frac{\alpha}{1-\alpha} \left( \frac{1}{\beta} + \alpha \right) \)

Proof. Plug the capital dynamics equation \( 9 \) and the equilibrium price conditions \( 2 \) in the insiders indirect utility function \( 3 \). Keeping only terms that depend on policy variables \( \tilde{l}_t \) and \( \tilde{l}_{t+1} \) gives \( 10 \). 

According to lemma \( 1 \) the representative insider objective function is independent of the physical capital stock. The unique relevant state variable in the optimal minimum wage policy choice of insiders is the current human capital stock \( E_t \). The parameter \( S \) can be interpreted as the intertemporal elasticity of substitution of human capital. Namely, an insider will accept to trade one unit of present human capital in exchange of \( S \) units of future human capital.

\[\text{With more general preferences or production function, majority rules among insiders choice should be applied. This would rule out any feasible analytical determination of the equilibrium policy and thus one should turn to numerical solutions of the type used in Krussel et al. (1996, 1999).}\]
Naturally, this price of human capital substitution rises as the agents become more impatient (lower \( \beta \)) and as capital becomes more productive (high \( \alpha \)), since in that case the value of current savings is higher. Tomorrow’s stock of human capital depends on today’s chosen minimum wage via the impact of employment opportunities on the human capital investment of the young. This objective function shows the main trade-off faced by current insiders: as wage laborer they rather benefit from a lower \( \bar{t} \) (labor market rigidity) to minimize today’s aggregate human capital \( H_t \) conditional on keeping their job, but as tomorrow’s capitalist they will rather benefit from a lower minimum wage (labor market flexibility) to foster human capital accumulation \( H_{t+1} \) so as to increase the returns of their savings.

We shall stress the need to adopt a particular equilibrium criterion that restricts the set of possible equilibria. In the unrestricted case, rational and forward looking insiders would play an intergenerational game with an infinite sequence of players and solve the following nested program:

\[
\begin{align*}
\max & \quad -S \ln H_t(\bar{t}_t; E_t) + \ln H_{t+1}(\bar{t}_{t+1}, \bar{t}_{t+1}) \\
\text{s.t.} & \quad E_{t+1} = E(\bar{t}_t), \text{ and } \bar{t}_t \in [0, 1] \text{ and } \bar{t}_{t+1} \in [0, 1] \\
\text{s.t.} & \quad \bar{t}_{t+1} = \arg \max -S \ln H_{t+1}(\bar{t}_{t+1}; E_{t+1}) + \ln H_{t+2}(E_{t+2}, \bar{t}_{t+2}) \\
\text{s.t.} & \quad E_{t+2} = E(\bar{t}_{t+1}), \text{ and } \bar{t}_{t+1} \in [0, 1] \text{ and } \bar{t}_{t+2} \in [0, 1] \\
& \ldots
\end{align*}
\]

An equilibrium will then be a sequence of policy functions \( \bar{t}_t(.), \bar{t}_{t+1}(.), \ldots, \bar{t}_{t+1}(., \ldots, \bar{t}_{t+1}(., \ldots, \bar{t}_{t+1}(., \ldots) \ldots) \) of past and future generations of insiders. The political equilibrium so obtained is a perfect Nash equilibrium where each middle-aged generation chooses the optimal minimum wage by fully discounting the effect it has on next period sequence of insiders choices. Models incorporating repeated voting with possible strategic interactions can usually only be solved numerically. Notably, in the context of redistributive taxation, this is the path followed by Krusell and Rios-Rull (1996, 1999) and more recently by Saint-Paul (2002). Rather, we shall first restrict to a subset of sub-game perfect equilibria namely the set of Markov perfect equilibria and rely on the methodological approach put forward by Hassler et al. (2003) to provide an analytical characterization for the set of equilibria. To our knowledge their approach has not been applied to the study of the dynamics of labor market institutions with their connections to the accumulation of physical and human capital, which are essential to growth. The next definition adapted from Hassler et al. (2003) specifies the equilibrium concept adopted:

**Definition 1** A Markov perfect political-economic equilibrium is:

A constant policy rule that for the current state of the economy gives the policy chosen by the critical voter, \( \bar{L} : [0, 1] \rightarrow [0, 1] \)

A private decision rule for human capital supply, \( E : [0, 1] \rightarrow [0, 1] \)

Such that the following functional equations hold

\[
\bar{L}(E_t) = \arg \max_{\{l_t\}} V(\bar{t}_t, \bar{t}_{t+1}, E_t, E_{t+1})
\]
s.t.
\[
\hat{E}_{t+1} = \hat{L}(E(\hat{E}_t))
\]
\[
E_{t+1} = E(\hat{E}_t) = x\hat{E}_t
\]
\[
\hat{I}_t = \hat{L}(E(\hat{E}_{t-1}))
\]

and a sequence of prices for capital and labor such that economic equilibrium relation on prices (Eq. 3) and on quantities (Eq. 7) hold \( \forall t = 0, \ldots, \infty \).

It follows that in a Markov equilibrium the set of utility-maximising policies depends only on the current state of the economy through a constant mapping. Hence, the constant policy rule implies that for any \( s \) if \( \hat{I}_{t+s} = \hat{L}(E(\hat{I}_{t+s-1})) \) then \( \hat{I}_t = \hat{L}(E(\hat{I}_{t-1})) \). Since only current insiders vote over the minimum wage and since they all have the same objective functions, the critical voter is a representative insider. In particular, there is no ex-post conflict between insiders since they all have the same investment opportunities, there is no room for strategic votes that will change the identity of the critical voter. Strategic voting is present in so far as current insiders are aware of, and integrate in their own policy choice, the influence they have on future policies that will be adopted. The optimal choice of minimum wage is worked-out in a general equilibrium context where all prices are endogenous and voters are forward looking and aware of the consequences of their policy on the future policy choices.

The rationality and forward looking assumptions imply that insiders take into account that the chosen minimum wage has an impact on tomorrow’s preferred minimum wage via the impact of today’s minimum wage on tomorrow’s state variable, that is \( E_{t+1} = E(\hat{I}_t) \). Hence, while myopic insiders do not consider strategic interactions, that is \( \frac{\partial I_{t+1}}{\partial I_t} = 0 \), rational and forward looking insiders do. We show below, in Proposition (3), that a steady state political-economic equilibrium with myopic voters is always a corner solution with either full employment or youths being all unemployed. Rather, successive choices of forward looking insiders may lead to full employment with no wage rigidity or to an interior solution with youth being partly unemployed. Since there is no conflict among insiders one may view the insiders choice of wage policy as that of a representative union whose objective is to maximise the remaining lifetime utility of its current members.\(^{12}\)

We assumed that the critical voter’s is always an insider. Assume instead that the critical voter is not an insider. With a constant population the three generations have the same size. The capitalist or retiree unambiguously vote for full employment so as to maximize his saving’s return. The choice of young is not that clear. Since they do not have access to capital market, they benefit from the minimum wage since incomes transferred to insiders allow them to work with more capital tomorrow. This effect of financial repression on accumulation is in the spirit Japelli and Pagano (1994) who developed a model where borrowing constraints on youths increase the steady state utility of all agents in the economy by fostering capital accumulation. We wish to point out here that with generational differences in access to capital market, higher minimum wages does not necessarily reduce youths welfare. Indeed, an equilibrium featuring wage rigidity and youth unemployment may even Pareto dominate an equilibrium with full employment and

\(^{11}\) See Saint-Paul and Verdier (1997), where agents have different investment opportunities such that some agents can free-ride on the effort of others by investing abroad.

\(^{12}\) We may assume that at the time the vote takes place young workers have not yet entered the labor market. Since the three generations are of the same size it suffices then to assume that senior workers are just slightly more powerful than capitalists to decide about labor market institutions.
that:

Proposition 2 characterizes the two possible steady state political-economy equilibria in terms of employment increment in human capital brought by a marginal increase in youths employment rate (next proposition states the possible equilibria configuration that arises as a function of the net has multiple steady states equilibria two of this equilibria are stable and one is unstable. The economy being partly unemployed. Full unemployment is never an equilibrium. When the economy converges either to a full employment equilibrium or to an equilibrium with youths may display either a unique or multiple steady state equilibria. When the steady state is unique and a changing intercept. The dynamics for minimum wage policy implies that the economy 3.2 The equilibrium wage policy We shall further restrict production and preference parameters such that interior solutions are possible equilibria outcome. We denote \( z = \frac{dE}{dt} \ast (\eta_u - \eta_c) = x \Delta \eta \), the net increment in future human capital brought by a marginal increase in youths employment rate. We assume that this value is bounded as follows: \((S - 1) < z \leq (S - 1) \frac{(1+\eta_u)}{\eta_u - (S - 1)}\) and that \( S - 1 < \eta_u < (S - 1) (S + 1)\). We derive in the appendix the equilibria for the full set of possible parameters values. The next proposition characterizes the equilibrium policy rule for this restricted set of parameters and the corresponding human capital dynamics.

Proposition 1 The equilibrium policy rule followed by insiders is:

\[
\bar{L}(E_t) = \bar{l}_t = \begin{cases} 
  aE_t + b & \text{if } \frac{1-a}{b} \left(1 - \frac{1-a}{b} - b\right) < E_t < \frac{1-a}{b} \left(1 - \frac{1-a}{b} - b\right) \\
  aE_t + b' & \text{if } \frac{1-a}{b} \left(1 - \frac{1-a}{b} - b\right) < E_t < \frac{1-a}{b} \left(1 - \frac{1-a}{b} - b\right) \\
  1 & \text{if } E_t > \frac{1-a}{b} \left(1 - \frac{1-a}{b} - b\right)
\end{cases}
\] (12)

The private decision rule follows the dynamics,

\[
E_{t+1} = E \left( \bar{L}(E_t) \right) = \begin{cases} 
  a x \left( E_t - E^* \right) + E^* + dx & \text{if } \frac{1-a}{b} \left(1 - \frac{1-a}{b} - b\right) < E_t < \frac{1-a}{b} \left(1 - \frac{1-a}{b} - b\right) \\
  a x \left( E_t - E^* \right) + E^* & \text{if } \frac{1-a}{b} \left(1 - \frac{1-a}{b} - b\right) < E_t < \frac{1-a}{b} \left(1 - \frac{1-a}{b} - b\right) \\
  x & \text{if } E_t > \frac{1-a}{b} \left(1 - \frac{1-a}{b} - b\right)
\end{cases}
\]

with \( a = \frac{\Delta \eta}{S - 1} \), \( b = \frac{\eta_c(z-(S-1))}{(S-1)(z+1)} > 0 \), \( b' = \frac{z \eta_u - S (1 + \eta_u)}{z (S - 1)} \), \( d = \frac{1 + \eta_u - z}{z (S - 1)} \) and \( E^* = \frac{b' x}{1 - ax} \)

Proof. See appendix. ■

The policy rule is piece wise linear with respect to the state variable \( E_t \): it has a constant slope and a changing intercept. The dynamics for minimum wage policy implies that the economy may display either a unique or multiple steady state equilibria. When the steady state is unique the economy converges either to a full employment equilibrium or to an equilibrium with youths being partly unemployed. Full unemployment is never an equilibrium. When the economy has multiple steady states equilibria two of this equilibria are stable and one is unstable. The next proposition states the possible equilibria configuration that arises as a function of the net increment in human capital brought by a marginal increase in youths employment rate \( z \) and characterizes the two possible steady state political-economy equilibria in terms of employment\(^\text{13}\)

Proposition 2 \( \forall S > 1 \) then there is a value \( z_h \in ((S - 1), c(S - 1)) \) with \( c = \frac{1+\eta_u}{1+\eta_u-S} > 1 \) such that:

- if \( z < S \) the economy converges to a unique equilibrium with youth unemployment (see fig. 3).
- if \( z > z_h \) the economy converges to a unique equilibrium with full employment (see fig. 2)

\(^\text{13}\)See the appendix for equilibria corresponding to all possible parameters values.
• if $S < z < z_h$ the economy has multiple equilibria, an unstable equilibrium and two stable equilibria, one with strictly positive youth unemployment and the other with full employment (see fig. 4).

![Figure 2: Flexible wage policy: high employment equilibrium ($z > z_h$)](image)

The core results of propositions [1] and [2] lies on the political strategic complementarity between successive generations of insiders’ choices. Namely, a current increase in minimum wages (unemployment) sets the ground for future increases, while a current decrease sets the ground for later decreases, thus creating a positive feedback between successive policy choices mediated through equilibrium prices on the capital and labor markets. To grasp the intuition for these strategic complementarities let’s consider the standard inverse labor demand schedule depicted in Figure 5. The wage schedule is a decreasing function of labor whose slope is an increasing function of current stock of capital: $\frac{\partial^2 W_t}{\partial L_t \partial K_t} < 0$ and $\frac{\partial W_t}{\partial K_t} < 0$. Assume that current insiders (period $t$) decide to increase wage rigidities, the marginal drop in youths employment rises insiders’ wage by $\overline{AB}$ in Figure 5. As a consequence the next period stock of physical capital is higher and the next period stock of insiders’ human capital is lower. Hence the period $t + 1$ wage schedule, $W_{t+1}(l_{t+1})$, is steeper than the current wage schedule meaning that for the same marginal increase in youths unemployment the wage gains for period $t + 1$ insider is now

![Figure 3: Rigid wage policy: unemployment trap](image)
higher than in previous period ($\overline{AB} > \overline{AB}$). Hence, if it was optimal for period $t$ to increase wage rigidities it is even more so for period $t+1$ insiders. Assume rather that period $t$ insiders decide to decrease wage rigidities (see Figure 6) such that youths employment rate rises. Current insiders income decreases by $\overline{CD}$ and consequently the next period’s stock of physical capital is lower, while the better labor market prospect for youths fosters their investments in human capital. These combined effects make the $t+1$ wage schedule still flatter. Hence, if it was optimal for the period $t$ insiders to have more wage flexibility it is even more so for the period $t+1$ insiders since the wage costs from a marginal increase in employment is lower ($\overline{CD} < \overline{CD}$).

By similar arguments, it can be shown in the $(R_{t+1}, L_{t+1})$ space that a lower youths employment rate decreases the interest cost of higher physical capital accumulation $\frac{\partial R_{t+1}}{\partial K_{t+1} \partial L_{t}} < 0$, and makes the adoption of high wage policy more likely.
Considering Eq. (3), higher youths unemployment has a direct and an indirect negative price effect, and a positive income effect on insiders' utility. The price effect works through the direct negative impact of higher physical capital accumulation on future interest rates. Higher youths unemployment has also an indirect negative effect, by lowering human capital accumulation it decreases next period returns on savings. The positive income effect is due to the substitutability between insiders and outsiders labor. When the sensitivity of human capital investments to youths employment rate, \( z \), is low (\( z < S \)), the indirect negative price effect is low, and the positive income effect overcome the direct price effect. Thus, for all possible states of the economy insiders prefer taxing indirectly outsiders, the gain in current income overcome the cost of lower future returns on savings.

Instead for a high responsiveness of human capital investments to youths unemployment rate (\( z > z_h \)), insiders optimal strategy to increase their remaining lifetime utility is to rise their savings returns. Hence in that case insiders care about the adverse effect that youths unemployment has on human capital accumulation. Lower wages increase current youths employment rates, decreases marginal educational costs and foster human capital accumulation. This strategy is adopted for all states of the economy when \( z \) is sufficiently high since in that case the indirect negative price effect always overcome the positive income effect.

Hence two growth strategies emerge. One strategy focused on physical capital accumulation with youths unemployment that act as a tax on youths and retiree income and redistribute toward savers. The other strategy is rather based on human capital accumulation with a higher rate of investment in human capital. Indeed, one can check that investment as share of GDP, \( \frac{I}{Y} \), rises when youths unemployment increases (that is when there are human capital depletion):

\[
\frac{I}{Y} = (1 - \alpha) \frac{L^i(E_t)^{\beta} W(k(E))}{LW(k(E))} = (1 - \alpha) \frac{\beta L^i(E_t)}{1 + \beta L(i(E_t), E_t)}
\]
and 
\[
\frac{\partial(I/Y)}{\partial(E)} < 0
\]

For intermediate values of \(z\) there exists an initial human capital endowment for insiders such that they are indifferent between one or the other strategy and the economy has multiple steady state equilibria. Starting from this initial stock, if one generation starts to increase wage rigidity so will do the next generations till the economy reaches a positive unemployment trap. The result is due to the forward looking behavior of insiders, the intuition is as follows: starting from \(E^*\) an increase in youth employment rate (increase in \(\tilde{I}_t\)) decreases insiders current income, however \(R_{t+1}\) increases both because next period stock of capital \(K_{t+1}\) is lower and because next period human capital is higher due to human capital accumulation and the complementarity between human and physical capital. This effect alone is not sufficient to generate multiple equilibria, if on top of that insiders rationally expect their vote and the next period insider’s vote to be strategic complement \(\frac{\partial R_{t+1}}{\partial I_t} > 0\), then they have still more incentives to increase \(I_t\), because the price effect (higher \(R_{t+1}\)) overcome the negative income effect (lower \(W_t\)). The economy starts then a growth strategy based on human capital accumulation and converges to a steady state equilibrium with full employment. Instead, starting from \(E^*\), an increase in youths unemployment raises current insiders income but decreases the future interest rate since the economy will be endowed with more physical capital and less human for using it. While this effect alone does not generate the multiplicity of equilibria adding the rational expectation of strategic policy complementarity, \(R_{t+1}\) will decrease further. The economy starts then a growth strategy based on physical capital accumulation and converges to a steady state equilibrium with the youths being persistently unemployed. It is worth to note that when multiple steady state equilibria exists the economy has a low employment trap to which corresponds a specific growth equilibrium path. The political unemployment trap corresponds to \(\frac{1}{a} \left( \frac{1-b}{ax} - b \right) < E_t < \frac{1}{a} \left( \frac{1-b}{ax} - b' \right)\). For this range of relatively low values of insiders’ human capital stock whatever are the next period rational employment rate expectations, the current insiders’ endowment of human capital is so low that the wage gain from choosing a low employment rate overcomes the interest rate loss. In turn the chosen low employment rate depresses human capital accumulation and sets the ground for the next generations of insiders to keep on choosing low level of youths employment rate (see the proof of Proposition 1).

To gain insights on the relevance of considering the forward looking behavior of policy choices, we consider next the optimal policy adopted by myopic rational insiders. Insiders are myopic in as much that they do not consider the influence they have on future policy preferences. Namely, myopic insiders consider \(\tilde{I}_{t+1}\) as a parameter, \(\tilde{I}_{t+1}^{my} = \tilde{I}_{t+1}\) with \(\frac{\partial I_{t+1}^{my}}{\partial I_t} = 0\), an interior policy choice for myopic insiders expected the employment rate \(\tilde{I}_{t+1}\) to prevail the next period is:

\[
\tilde{I}_{t+1}^{my}(E_t, \tilde{I}_{t+1}) = \frac{\Delta \eta}{S-1} E_t + \frac{\eta_e (z - S) - S \tilde{I}_{t+1}^{my}}{(S-1) z}
\]

where \(\tilde{I}_{t+1}^{my}\) is the period \(t + 1\) rational expectations on future policy. The rationality of expectations imposes that expectations should be monotonic, that is if \(\tilde{I}_t > \tilde{I}_{t+1}^{my} = \tilde{I}_{t+1}\) then \(\tilde{I}_t^{my} > \tilde{I}_{t+2}^{my}\), while if \(\tilde{I}_t^{my} < \tilde{I}_{t+1}^{my} = \tilde{I}_{t+1}\) then \(\tilde{I}_t^{my} > \tilde{I}_{t+2}^{my}\). The intuition is the same as the one that we have grasped in Figures (5) and (6). The next proposition characterizes the myopic steady state equilibrium outcome:

\[\text{14 The details are provided in the Appendix in the proof of proposition } 3\]
Proposition 3

The steady state Myopic Equilibrium policy is:

\[
\tilde{\kappa}_{my} = \begin{cases} 
1 & \text{if } z > S \\
0 & \text{if } z \leq S
\end{cases}
\] (13)

Proof. See Appendix.

Intuitively if the gains from higher human capital accumulation brought by a marginal increase in current employment rate \(z\) outweigh the quantity at which insiders are indifferent between current and future increases \(S\), then the successive choices of insiders will drive the economy to full employment. The converse holds for \(z < S\). The discrepancy with the forward looking (FL) decisions is due to the fact that FL insiders can free ride on the decisions taken by the next period insiders when they decide to vote for a decreasing minimum wage profile. On one side an insiders wishing to increase youths employment rate find it more profitable if they expect next period insiders to react by increasing their own period employment rate (for the same wage loss, future interest rates gains are higher). This explains the non linearity of the wage policy. Myopic insiders can not rely on this complementarity of behavior. Accordingly myopic behavior should lead to slower convergence to full employment than forward looking behavior does. On the other side FL insiders that choose a high minimum wage profile bears part of the cost of the behavior of future generations of insiders. Indeed since those insiders will react by increasing their own period minimum wage, for the same positive income effect (wage gain) insiders have to incur a higher negative price effect (interest rates loss). Consequently the forward looking insiders minimum wage policy is in some sense ”more sluggish” downward than it is upward. This explains that there exist steady state equilibrium with myopic insider displaying full unemployment for the young while such a steady state does not exist with FL insiders. Moreover, since equilibria with myopic insiders have unique steady states our multiplicity result is clearly driven by the FL behavior of insiders.

3.3 Steady state equilibria

The next proposition gives the equilibrium prices and quantities for both steady state political-economy equilibria when insiders are forward looking and compares them.

Proposition 4 For parameters value such that there is multiple steady state, each steady state is associated to a different equilibrium growth path.

At the full employment steady state political economy equilibrium (squee) aggregate capital \(K_{fe}^*\), the capital labor ratio \(k_{fe}^*\) and aggregate output \(Y_{fe}^*\), are the following:

\[
K_{fe}^* = C \left( \frac{L^2(x)}{H(1)} \right) \frac{1}{\alpha x} H(1)
\]

\[
k_{fe}^* = C \left( \frac{L^2(x)}{H(1)} \right) \frac{1}{\alpha x}
\]

\[
Y_{fe}^* = C^n \left( \frac{L^2(x)}{H(1)} \right) \frac{n}{\alpha x} H(1)
\]

At the youths unemployment steady state political economy equilibrium (suue) aggregate capital \(K_{yu}^*\), the capital labor ratio \(k_{yu}^*\) and aggregate output \(Y_{yu}^*\), are the following:

\[
K_{yu}^* = C \left( \frac{L^2(x)}{H(1)} \right) \frac{1}{\alpha x} H(1 - \frac{b}{ax})
\]

\[
k_{yu}^* = C \left( \frac{L^2(x)}{H(1)} \right) \frac{1}{\alpha x}
\]
\[ Y_{yu}^* = C^\alpha \left( \frac{L^2(\frac{1-b}{H(\frac{a}{\alpha})})}{H(\frac{1-b}{\alpha})} \right)^{-\frac{\alpha}{\beta}} H(\frac{1-b}{\alpha}) \]

where \( C \) is a constant, \( C = \frac{\beta}{\alpha + 1} (1 - \alpha) A \).

The following inequalities hold at the steady state:

\[
\begin{align*}
    k_{yu}^* & > k_{fe}^* \\
    K_{fe}^* & > K_{yu}^* \\
    Y_{fe}^* & > Y_{yu}^*
\end{align*}
\]

**Proof.** see Appendix. □

According to this proposition while productivity is higher in the low employment equilibrium aggregate output is lower as well as aggregate saving rate. Indeed, the higher productivity is due to higher capital deepening resulting from lower labor utilization. These results are broadly consistent with empirical studies that compares some European countries to the U.S.. These studies show that until mid-1990s productivity in some European countries (notably France and Germany) was higher than in the US while per capita income remains lower (see Beaudry and Collard 2003; Gordon, 1995). It is interesting to note that while youths unemployment increases insiders’ income and the short run economy saving rate which may in the short run increase growth, in the long run aggregate saving is lower in the economy with unemployment, because of the adverse impact it has on human capital accumulation. This is a similar argument as the one developed by Gordon (1995) whereby the trade-off between productivity and labor is only temporary as higher unemployment decreases workers available income and lowers aggregate savings. Hence the productivity lead of Europe is only temporary since it drives capital depletion in the long run. In our model unemployment raises saving in the short run since it redistribute income toward groups of workers with higher propensity to save, but in the long run through its negative impact on human capital accumulation, unemployment is also detrimental to physical capital accumulation. In Appendix B I show, within an endogenous growth model based on a physical capital externality \( a la \) Romer (1989), that in this case steady state growth of productivity is also lower in the high unemployment equilibrium, suggesting that the employment-productivity trade-off may be only temporary.

Some simple equilibrium comparative statics are interesting. The first refers to the impact of an increase in \( z \) and the other to the impact of a change in \( S \). An increase in \( z \) can be assimilated to a skill biased technical change, that raises the efficiency of skilled labor compared to the unskilled. We assume that \( \Delta \eta = \eta_e - \eta_u \) remains constant, while \( \frac{\eta_u}{\eta_e} \) increases. Over the 1990s and 1980s inequality between skilled and unskilled workers have risen in both absolute and relative terms (Caselli, 2000). Without loss of generality I assume that relative inequality \( \frac{\eta_u}{\eta_e} \) increases while absolute inequality remains constant. Depending on whether they are in the high or in the low employment equilibrium, economies will adapt very differently to such this increase in returns to skills. In the high employment equilibrium following a marginal increase in \( z \), human capital investment increases and, due to higher capital accumulation, productivity \( (k_{fe}^*) \) rises while the economy remains at the full employment equilibrium. Instead, in the low

\[15\]At least till the mid 1990’s, indeed since then hourly productivity in US catch that with europe. The most plausible explanation put forward is that of a lag effect on productivity of investment on ICT made earlier in the US (ref....).

\[16\]We can think of an experienced biased technological change, which according to Caselli 2014 may explain that experience premium has not failed despite the increase in the share of more experienced workers.
employment equilibrium, the wage schedule is now steeper than it was before, meaning that the wage gains of future insiders from increasing wage rigidity is now higher. Hence a moderate increase in $z$ has a negative impact on youths employment rate.

An increase in $S$ may be due to a change in the capital share of income ($\alpha$). In the high unemployment equilibrium an increase in $S$ rises the equilibrium employment and human capital. Indeed due to a higher marginal productivity of capital, future human capital worth more because the loss of interest rate is higher. In the high employment equilibrium a marginal increase in $\alpha$ has virtually no impact on employment and human capital accumulation.

The general lesson that we want to stress through these straightforward comparative statics is that depending on their initial steady state equilibrium and human capital stock economies adapt very differently to the same shocks when one consider endogenous labor market institutions. This may explain why some economies failed to take advantage of technological changes to rise their stock of human capital. In our model this is due to the uncoordinated behavior of successive generations of insiders. Following the same raise in returns to skill some economies adapt by accumulating human capital while in others higher youth unemployment may dampen incentives brought by higher skill premium and human capital may fail to adjust.

In the model we have been quite agnostic about the specific institution through which outsiders labor supply is rationed, one may think that is takes the form of a minimum wage. However we thing that the argument is more general and remains valid for any other policy that increases the relative cost of hiring an outsider compared to an insider as firing and hiring costs. In the model the firing cost has been set to infinity to focused on wage policy affecting outsiders. Moreover we have assumed that unemployment creates human capital depletion by increasing the consumption costs of training. One may consider a model where, as in Pissarides (1992), unemployment creates human capital depletion independently of its impact on training per-se. Indeed, it is a well known fact of Mincerian wage regression that experience is positively correlated with productivity (Topel, 1992). Hence if at any age and education levels insiders in the first periods on their labor market experienced unemployment spells then on average they should be endowed with less human capital than the same insiders that instead experienced high employment.

## 4 Conclusion

In most countries unemployment fell disproportionately on youths segment of the labor market. Those have been considered as outsiders in our model. Youth is precisely a life period during which critical human capital is acquired either in school or directly on the labor market. In the context of a three period OLG model we have shown that differences in labor market institutions, labor productivity and per capita income, can be interpreted as politico-economic equilibria of countries with different stock of initial human capital and otherwise identical fundamentals. The optimal wage policy adopted by rational and forward looking insiders face an intergenerational trade-off aimed at maximizing their remaining lifetime utility. In countries with high initial human capital, insiders adopt a flat wage profile and follow a human capital driven growth along the transition to a steady state characterized by lower labor productivity and higher income per capita. In countries with lower initial human capital successive generations of insiders adopt a steeper wage profile and the economy has a physical capital driven growth. Unemployment experienced by successive generations of youths creates human capital depletion, along the
transition to a steady state characterized by higher labor productivity and lower income per capita.

The model contribute to understand the endogeneity of labor market institutions and their future prospect in relation to their effect on different generations of workers, labor productivity and accumulation of human and physical capital. The Overlapping Generation Structure of the model should allow fruitful extensions. In the Appendix B the model is extended to account for endogenous growth. We show that the steady state productivity growth is higher in the high employment equilibrium such that the employment productivity trade-off may be only temporary.

Another important issue that may be handled in this framework is the consideration of the complementarity/substitutability of labor market institutions with other institutions related to the welfare state together with their long run effect on growth and productivity. An important one that should be considered is the effect of a pension system that will break the link between one actual and future income which is rather a characteristic of the fully funded system considered here. The parameter of the pension system is likely to be non neutral with respect to the dynamics of wage rigidities. Population growth or migration as an exogenous source of human capital accumulation is worth to be considered. In our model the introduction of population growth or exogenous migration should make wage rigidities more costly for insiders since they increase the returns to their future capital income. Empirically this predicts that wage rigidities should be less likely in economies with higher population growth. We have assumed that workers become insiders as they aged independently of their unemployment experience. If unemployment experience bears on the probability to be insiders than this will be equivalent to a reduction in insiders future stock of human capital which should increase the cost of wage rigidities. Lastly the model is a closed economy model. Capital mobility within an heterogeneous economic union should lower the capability of insiders to increase their wage income and should improve youths labor prospect. Things becomes more intricate if one allows for cooperative or non cooperative behavior of foreigners choice of wage policies such as a common minimum wage, this should raise issues akin to those considered in the fiscal competition literature.

The empirical literature on labor market institutions has mostly been concerned with the impact of institutions on employment and income distribution, by emphasizing its role in the allocation of individuals to jobs. Medium to long run effects of labor market institutions and its impact on youths employment opportunities have received yet little attention (Topel, 1999). Empirically this paper suggests that labor market institutions affecting youth unemployment prospect have important long run effect on growth and productivity. Hence to fully understand their impact from the medium to the long run this paper points out that it is crucial to consider, and control for, their short run redistributive effects across different generations of workers.

References


APPENDIX

A Proofs

Proposition (1)

The first step in the quest of Markov equilibria is to guess the period \( t + 1 \) optimal rule \( l_{t+1} = \tilde{L}(E(\tilde{l}_t)) \) and check that this is also the optimal rule followed by current insiders. This optimal rule is characterized in the following lemma:

Lemma 2

I note \( z \equiv \frac{dE}{dE} (\eta_e - \eta_u) \equiv x \Delta \eta > 0 \) an index measuring the net increase in human capital following a marginal increase in youths employment rate, hence this is a measure of the sensitivity of human capital supply to employment, and \( S = \frac{\alpha}{1-\alpha} \left( \frac{1}{\beta} + \alpha \right) \)

- If \( z > \frac{1 + \eta_u}{\eta_u - (S - 1)} (S - 1) \)
  \[ l_{t+1} = 1 \] (14)

- If \( (S-1) < z \leq \frac{1 + \eta_u}{\eta_u - (S - 1)} (S - 1) \)
  \[ \tilde{l}_{t+1} = \tilde{L}(E(\tilde{l}_t)) = \begin{cases} 1 & \text{if } \tilde{l}_t > \frac{1-b}{a \tilde{x}} \\ \text{ax} \tilde{l}_t + b & \text{if } \tilde{l}_t \leq \frac{1-b}{a \tilde{x}} \end{cases} \] (15)
  \[ \text{with } a > 1 \text{ and } b > 0 \] (16)

- If \( \tilde{z} < z \leq (S-1) \) where \( \tilde{z} : (\tilde{z})^2 + \tilde{z}(1 + \eta_s) - \eta_s (S - 1) = 0 \)
  \[ \tilde{l}_{t+1} = \tilde{L}(E(\tilde{l}_t)) = \begin{cases} \text{ax} \tilde{l}_t + b & \text{if } \tilde{l}_t > \frac{-b}{a \tilde{x}} \\ 0 & \text{if } \tilde{l}_t \leq \frac{-b}{a \tilde{x}} \end{cases} \] (17)
  \[ \text{with } a < 1 \text{ and } b < 0 \] (18)

- If \( z \leq \tilde{z} \)
  \[ l_{t+1} = 0 \]

and \( a = \frac{\Delta \eta}{S-1} > 0 \) and \( b = \frac{\eta_s (z - (S-1))}{(S-1)(z+1)} \)

Proof. First step: guess a functional form with unknown parameters for the policy function next period, in a Markov equilibrium this variable depends only on the current state. Here the state variable is the number of skilled workers \( E_t \). Then, verify that the same function determines present choices.

With restriction to Markov equilibria we just need to anticipate the vote one period ahead. We can guess a linear policy rule, assume that: \( \tilde{L}(E_t) = aE_{t+1} + b \) replacing \( E_{t+1} \) by the private rule \( E(l_t) = xl_t \) one has:

\[ \tilde{l}_{t+1} = \tilde{L}(E_t) = ax \tilde{l}_t + b \] (19)

Hence we can plug in the critical voter choice the optimal policy rule assumed, and the private policy rule which gives the following problem to solve:

\[ \max_{\tilde{l}_t} -\alpha(1 + \alpha \beta) \ln \left( \tilde{l}_t + \eta_n + E_t \Delta \eta \right) + \beta(1 - \alpha) \ln \left( ax \tilde{l}_t + b + \eta_n + E_{t+1} \Delta \eta \right) \]

s.t. \( E_{t+1} = E(l_t) = xl_t \) and \( \tilde{l}_t \in [0, 1] \)

\[ \text{max} -\alpha(1 + \alpha \beta) \ln \left( \tilde{l}_t + \eta_n + E_t \Delta \eta \right) + \beta(1 - \alpha) \ln \left( ax \tilde{l}_t + b + \eta_n + E_{t+1} \Delta \eta \right) \]

s.t. \( E_{t+1} = x \tilde{l}_t \) and \( \tilde{l}_t \in [0, 1] \)

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\[ FOC: \]
\[
\frac{\ell_t + \eta_t + E_t \Delta_t}{\alpha x \ell_t + b + \eta_t + x \ell_t \Delta_t} = \frac{x(a + \Delta_t)}{a x \ell_t + b + \eta_t + x \ell_t \Delta_t}
\]
with \( S = \frac{a(1+\alpha \beta)}{\beta(1-\alpha)} > 1 \) (by assumption a sufficient condition is \( \alpha > \beta \)), this gives:

\[
\tilde{\ell}_t = \frac{\Delta_t}{S-1} \] 

with \( S = \frac{1}{1+u} \).

Second step: identify coefficients

\[
a = \frac{\Delta_t}{S-1}, \quad b = \frac{x \eta_t (a+\Delta_t) - S(b+\eta_t)}{x(a+\Delta_t)(S-1)}
\]

Hence, since \( \Delta_t > 0 \) and applying the constraint \( \tilde{\ell}_t \in [0,1] \) one has that:

\[
\tilde{\ell}_{t+1} = \tilde{L}(E_t) = \begin{cases} 
1 & \text{if } \frac{\tilde{\ell}_t}{a x \ell_t + b} \leq \frac{1-b}{\alpha x} \\
0 & \text{if } \frac{\tilde{\ell}_t}{a x \ell_t + b} > \frac{1-b}{\alpha x}
\end{cases}
\]

However not all those choices are mutually consistent for all parameter values, denote \( z = x \Delta_t > 0 \)

- \( \frac{1-b}{\alpha x} < 1 \) iff

\[
k(z) = z^2 - z [(S-1) - (1+\eta_a)] - (S-1)(1+\eta_a) \geq 0
\]

with a root: \( z_1 = (S-1) \)

\[
\text{hence } \frac{1-b}{\alpha x} \leq 1 \text{ for all } z \geq (S-1).
\]

- \( \frac{1-b}{\alpha x} > 0 \) iff

\[
b < 1 \Leftrightarrow z < (S-1) \frac{(1+\eta_a)}{\eta_a - (S-1)} = (S-1) c
\]

with \( c = \frac{1+\eta_a}{1+\eta_a-S} > 1 \), hence

\[
\frac{1-b}{\alpha x} \in (0,1) \Leftrightarrow (S-1) < z \leq (S-1) c
\]

- \( \frac{-b}{\alpha x} > 0 \) iff \( b < 0 \Leftrightarrow z < (S-1) \)

- \( \frac{-b}{\alpha x} \leq 1 \) iff

\[
h(z) = z^2 + z(1+\eta_a) - \eta_a (S-1) \geq 0
\]

with the positive root:

\[
z = \frac{(1+\eta_a) + \sqrt{(1+\eta_a)^2 + 4\eta_a(S-1)}}{2}
\]

then \( \frac{-b}{\alpha x} \leq 1 \) for all \( z > z \). Note that \( h(S-1) > 0 \), hence \( (S-1) > z \). The quantity \( \frac{-b}{\alpha x} \) is a valid candidate if the following conditions are met:

\[
\frac{-b}{\alpha x} \in (0,1) \Leftrightarrow z < z \leq (S-1)
\]
• If \( z < \bar{z} \) then \( \frac{1-b}{ax} > \frac{b}{ax} > 1 \) while if \( z > (S-1)c \) then \( \frac{b}{ax} < \frac{1-b}{ax} < 0 \).

Henceforth I will focus on cases where interior solutions can emerge that is \( (S-1) < z \leq (S-1)c \) and \( z < \bar{z} \leq (S-1) \).

For extreme values of \( z \), that is either a high or a low productivity gap between both workers’ types, the unique rational expectation regarding next period youths employment rate is a corner solution. If the human capital loss associated to unemployment is high then the increase in returns to capital outweighed the cost of lower current wage income and insiders choose full employment. For intermediate ranges of this loss, next period youths employment rate depends on current period choice of policy via its impact on next period state variable \( E_t \), this is the essence of Markov equilibria. Once \( t_{t+1} \) is determined one can turn to the determination of current period labor market policy \( \tilde{t}_t \). We will focus on the case where interior solutions can emerge in steady state equilibrium, that is \( S-1 < z \leq (S-1)c \). We prove next the core of proposition (II):

**Proof.** Assume \( (S-1) < z \leq (S-1)c \)

• Then the optimal policy rule for \( t+1 \) insiders is:

\[
\tilde{t}_{t+1} = \tilde{L}(E(\tilde{t}_t)) = \begin{cases} 
  \frac{1}{ax\tilde{t}_t + b} & \text{if} \quad \tilde{t}_t > \frac{1-b}{ax} \\
  \frac{1}{\tilde{t}_t + \frac{b}{ax}} & \text{if} \quad \tilde{t}_t \leq \frac{1-b}{ax}
\end{cases} \tag{28}
\]

The insider objective function is not differentiable at \( \tilde{t}_t = \frac{1-b}{ax} \). Hence we need to define \( \hat{V}^a \) and \( \hat{V}^b \) as follows:

\[
\hat{V}^a = \max_{\tilde{t}_t \in (\frac{1-b}{ax},1]} -\alpha(1 + \alpha\beta) \ln (\tilde{t}_t + \eta_u + E_t\Delta_\eta) + \beta(1-\alpha) \ln (1 + \eta_u + E_{t+1}\Delta_\eta) \\
\text{st } E_{t+1} = xl_t \quad \text{and} \quad \tilde{L}(E_t) = 1
\]

for an interior solution the optimal policy choice in the set \( (\frac{1-b}{ax},1] \) is:

\[
\tilde{t}_t^a = aE_t + b'
\tag{29}
\]

with \( b' = b - \frac{S-1}{S-1} \frac{1+\eta_u}{ax\Delta_\eta(\eta_u)} < b \) and \( b' < 0 \) and

\[
\hat{V}^a = \begin{cases} 
  -\alpha(1 + \alpha\beta) \ln (1 + \eta_u + E_t\Delta_\eta) + \\
  \beta(1-\alpha) \ln (1 + \eta_u + x\Delta_\eta) & \equiv \hat{V}^{a,corr2} \quad \text{if} \quad E_t > \frac{1-b'}{a} \\
  -\alpha(1 + \alpha\beta) \ln (\tilde{t}_t^a + \eta_u + E_t\Delta_\eta) + \\
  \beta(1-\alpha) \ln (1 + \eta_u + x\Delta_\eta) & \equiv \hat{V}^{a,int} \quad \text{if} \quad \frac{1}{a} (\frac{1-b}{ax} - b') < E_t < \frac{1-b'}{a} \\
  -\alpha(1 + \alpha\beta) \ln (\frac{1-b}{ax} + \eta_u + E_t\Delta_\eta) + \\
  \beta(1-\alpha) \ln (1 + \eta_u + x\Delta_\eta) & \equiv \hat{V}^{a,corr1} \quad \text{if} \quad E_t < \frac{1}{a} (\frac{1-b}{ax} - b')
\end{cases}
\]

and:

\[
\hat{V}^b = \max_{\tilde{t}_t \in (0,\frac{1-b}{ax}]} -\alpha(1 + \alpha\beta) \ln (\tilde{t}_t + \eta_u + G_t\Delta_\eta) + \beta(1-\alpha) \ln (\tilde{t}_{t+1} + \eta_u + G_{t+1}\Delta_\eta) \\
\text{st } G_{t+1} = xl_t \quad \text{and} \quad \tilde{L}(G_t) = ax\tilde{t}_t + b
\]
for an interior solution the optimal policy choice in the set \((0, \frac{1-b}{ax})\) is:

\[
\hat{t}_t^b = aE_t + b
\]

then,

\[
\hat{V}^b = \begin{cases} 
-a(1+\alpha\beta)\ln\left(\frac{1-b}{ax} + \eta_u + E_t\Delta \eta\right) + \\ \beta(1-\alpha)\ln\left(1 + \eta_u + \frac{1-b}{ax}\Delta \eta\right) + \\ -a(1+\alpha\beta)\ln\left(\hat{t}_t^b + \eta_u + E_t\Delta \eta\right) + \\ \beta(1-\alpha)\ln\left(t_{t+1}(\hat{t}_t^b) + \eta_u + x\hat{V}_t^b\Delta \eta\right) 
\end{cases}
\]

\[
\equiv \hat{V}_{\text{corr}}^{b,\text{opt}} \quad \text{if} \quad E_t \geq \frac{1}{a} \left(\frac{1-b}{ax} - b\right) \\
\equiv \hat{V}_{\text{int}}^{b,\text{opt}} \quad \text{if} \quad E_t < \frac{1}{a} \left(\frac{1-b}{ax} - b\right)
\]

Note first that since \(a > 0\) and \(b > 0\), \(\hat{t}_t^b > \hat{t}_t^a\), whenever \(\hat{t}_t^b\) is an interior solution \(\hat{t}_t^a\) is a corner solution and \(\hat{V}_{\text{corr}}^{b,\text{opt}} = \hat{V}_{\text{corr}}^{a,\text{opt}}\). Hence we have the following result from which we derive the dynamic of minimum wage as well as that of \(E_t\), by applying the law of motion \(E_{t+1} = x\tilde{L}(E(\hat{t}_t))\).

- If \(E_t > \frac{1-b'}{a} \) then \(\hat{t}_t^a > 1 \Rightarrow \hat{t}_t^a = 1\) and \(\hat{t}_t^b = \frac{1-b}{ax}\) hence \(\hat{V}^b = \hat{V}_{\text{corr}}^{b,\text{opt}} = \hat{V}_{\text{corr}}^{a,\text{opt}}\) and \(\hat{V}^a = \hat{V}_{\text{corr}}^{a,\text{opt}}\), but then one see that \(\hat{V}_{\text{corr}}^{a,\text{opt}} > \hat{V}_{\text{corr}}^{b,\text{opt}} = \hat{V}_{\text{corr}}^{a,\text{opt}}\) since \(\frac{\partial V}{\partial t_i} > 0\) for \(\hat{t}_t < \hat{t}_t^a\), and \(\tilde{L}(G_t) = 1\).

- If \(\frac{1}{a} \left(\frac{1-b}{ax} - b\right) < \frac{1}{a} \left(\frac{1-b}{ax} - b'\right) < E_t < \frac{1-b'}{a} \) then \(\hat{t}_t^b = \frac{1-b}{ax}\) and \(\hat{t}_t^a \in \left(\frac{1-b}{ax}, 1\right)\). It implies that \(\hat{V}^a = \hat{V}_{\text{int}}^{a,\text{opt}}\) and \(\hat{V}^b = \hat{V}_{\text{corr}}^{b,\text{opt}} = \hat{V}_{\text{corr}}^{a,\text{opt}}\) and since \(\hat{V}_{\text{int}}^{a,\text{opt}} > \hat{V}_{\text{corr}}^{a,\text{opt}}\) the optimal policy is \(\tilde{L}(E_t) = \hat{t}_t^a = \hat{t}_t^b = aE_t + b\).

- If \(\frac{1}{a} \left(\frac{1-b}{ax} - b\right) < E_t < \frac{1}{a} \left(\frac{1-b}{ax} - b'\right)\), \(\hat{t}_t^a = \frac{1-b}{ax}\) and \(\hat{t}_t^b = \frac{1-b'}{ax}\) then \(\hat{t}_t^b = \hat{V}_{\text{corr}}^{b,\text{opt}} = \hat{V}_{\text{corr}}^{a,\text{opt}}\) the optimal policy is then \(\tilde{L}(E_t) = \frac{1-b}{ax}\).

- If \(E_t < \frac{1}{a} \left(\frac{1-b}{ax} - b\right)\), \(\hat{t}_t^a\) is an interior solution and \(\hat{t}_t^b = \frac{1-b}{ax}\) then optimal policy is \(L(G_t) = \hat{t}_t^a = aE_t + b\).

We deduce the following dynamics for the optimal policy rule:

\[
\tilde{L}(E_t) = \begin{cases} 
1 \quad \text{if} \\ aE_t + b' \quad \text{if} \quad \frac{1}{a} \left(\frac{1-b}{ax} - b\right) < E_t < \frac{1-b'}{a} \\ aE_t + b \quad \text{if} \quad E_t < \frac{1}{a} \left(\frac{1-b}{ax} - b\right)
\end{cases}
\]

Applying the law of motion \(E_{t+1} = E(\tilde{L}(E_t)) = x\tilde{L}(E_t)\) we deduce the dynamic of human capital accumulation:

\[
E_{t+1} = \begin{cases} 
x(E_t - E^*) + E^* + cx \quad \text{if} \quad E_t < \frac{1}{a} \left(\frac{1-b}{ax} - b\right) \\ \frac{1-b}{a} \quad \text{if} \quad \frac{1}{a} \left(\frac{1-b}{ax} - b\right) < E_t < \frac{1}{a} \left(\frac{1-b}{ax} - b'\right) \\ ax(E_t - E^*) + E^* \quad \text{if} \quad \frac{1}{a} \left(\frac{1-b}{ax} - b'\right) < E_t < \frac{1}{a} \left(\frac{1-b'}{ax} - b\right) \\ x \quad \text{if} \quad E_t > \frac{1}{a} \left(\frac{1-b'}{ax} - b\right)
\end{cases}
\]

We can have two economies with exactly the same stock of human capital, \(\eta_u + E_t\Delta \eta\), the one whose human capital is more sensitive to employment is more likely to converge toward a
full employment equilibrium. Two economies that are identical but have slightly different initial level of human capital (in the neighborhood of $E^*$) the economy with $E < E^*$ converge toward a steady state equilibrium with employment, while the economy with $E > E^*$ converge toward an economy with full employment.

**Proof. Set of equilibria in proposition 4**

- According to the dynamics in (31), a sufficient condition for multiple equilibria is:

  $$
  \frac{1}{a} \left( \frac{1 - b}{ax} - b' \right) < E^* = \frac{b'x}{1 - ax} < x \text{ and } b' < 0
  $$

  where $E^*$ is the fix point of the recurrent equation $E_{t+1} = axE_t + b'x$. Depending on parameters’ value three possible equilibrium regimes can emerge, two entails a unique steady state and one has multiple steady states:

  - $E^* < \frac{1}{a} \left( \frac{1 - b}{ax} - b' \right)$ then we have a unique equilibrium with full employment
  - $\frac{1}{a} \left( \frac{1 - b}{ax} - b' \right) < E^* = \frac{b'x}{1 - ax} < x$ then we have multiple equilibrium one with full employment and the other with unemployment
  - $E^* = \frac{b'x}{1 - ax} > x$ then we have a unique equilibrium with unemployment

  We remind that $E^* = \frac{b'x}{1 - ax}$ is the point corresponding to the intersection of the third segment $E_t = ax + b'$ (see Figures 2, 3, 4 in the text) and the 45° line.

  The right hand side condition for multiple equilibria is:

  $$
  \frac{b'x}{1 - ax} < x \Leftrightarrow b' + ax > 1
  $$

  Replacing $b'$ and $ax$ by their respective values and with $z > S - 1$ the condition is:

  $$
  b' + ax > 1
  \Leftrightarrow H(z) = (z - S)(Z + \eta_u + 1) > 0
  \Leftrightarrow z > S
  $$

  such a $z$ exists iff $S < c(S - 1) \Leftrightarrow \eta_u < (S - 1)(S + 1)$

  The condition on the left side of $G^*$ is $G^* > \frac{1}{a} \left( \frac{1 - b}{ax} - b' \right)$

  $$
  E^* > \frac{1}{a} \left( \frac{1 - b}{ax} - b' \right) = E \Leftrightarrow
  K(z) = (S - 1)z^2 - z(1 + (S - 1)((S - 1) - \eta_u)) - (1 + \eta_u)(S + (S - 1)^2) < 0
  $$

  if $\eta_u > (S - 1)S$, $K(z) < 0$ then $\forall z \in [(S - 1), c(S - 1)], E^* > E$

  if $\eta_u \leq (S - 1)S$, $E^* > E \Leftrightarrow z < z_h \in ((S - 1), c(S - 1))$ and $S < z_h$ since $K(S) < 0$ and $K(c(S - 1)) > 0$.

  Depending on the value of $\eta_u$ and $z$ the following equilibria are possible
• If $(S - 1) < \eta_u < (S - 1) (S - 1) (S + 1)$
  - if $S - 1 < z < S < z_h < c(S - 1)$ then $E^* > x > E$ and the economy converges to an equilibrium with unemployment and $E^* = \frac{1 - b}{a}$.
  - if $S < z < z_h < c(S - 1)$ then $E < E^* < x$ and the economy has multiple equilibria, one unstable equilibrium and two stable equilibria, one with full employment and the other with a positive level of unemployment rate for the young.
  - if $S < z_h < z < c(S - 1)$ then $E^* < E < x$ and the economy converges to a unique full employment equilibrium.

• If $(S - 1) S < \eta_u < (S - 1) (S + 1)$
  - if $S - 1 < z < S < c(S - 1)$ then $E^* > x > E$ and the economy converges to an equilibrium with unemployment and $E^* = \frac{1 - b}{a}$.
  - if $S < z < c(S - 1)$ then $E < E^* < x$ has multiple equilibria, one unstable equilibrium and two stable equilibria, one with full employment and the other with positive youths unemployment rate.

• If $(S - 1) S < (S - 1) (S + 1) < \eta_u$ the economy converges to an equilibrium with unemployment and $E^* = \frac{1 - b}{a}$.

The following proposition sum-up the result:

**Proposition 5** \forall S > 1 there is a $\eta_e = (S - 1) (S + 1)$ such that if $\eta_e < \eta_e$ then there exist a $z_h$ and a $z_l$ with $z_h > z_l$ and $z_h, z_l \in ((S - 1), c(S - 1))$ such that:

- if $z < z_l$ the economy converges to a unique equilibrium with unemployment
- if $z > z_h$ the economy converges to a unique equilibrium with full employment
- if $z_l < z < z_h$ the economy has multiple equilibria, an unstable equilibrium and two stable equilibria, one with unemployment and the other with full employment.

**Equilibria with myopic insiders:**

**Proof.** Proposition 3

Myopic insiders take the next period expected employment rate as a parameter $\tilde{l}_{t+1}$ and choose $\tilde{l}_{t}^{my}$ such that:

$$\tilde{l}_{t}^{my} = \arg \max_{l_t \in [0, 1]} V^{my}(\tilde{l}_t; E_t; \tilde{l}_{t+1}^a) = -S \ln \left(\tilde{l}_t + \eta_e + E_t \Delta_n\right) + \ln \left(\tilde{l}_{t+1}^a + \eta_e + E_{t+1} \Delta_n\right)$$

$$\text{st } E_{t+1} = x \tilde{l}_t$$

The FOC is:

$$\frac{S}{\tilde{l}_t + \eta_u + E_t \Delta_n} = \frac{z}{\tilde{l}_{t+1}^a + \eta_u + E_{t+1} \Delta_n}$$

(33)

$$\tilde{l}_{t}^{my}(E_t, \tilde{l}_{t+1}^a) = \frac{\Delta \eta}{S - 1} E_t + \frac{\eta_u (z - S) - S \tilde{l}_{t+1}^a}{(S - 1) z}$$

One can check that

$$a E_t + b' < \tilde{l}_{t}^{my} < a E_t + b$$

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• We shall establish the following:
  (i) if agents are myopic there is no interior solution with steady state solution, hence steady state solutions with myopic behavior are corner solutions
  (ii) corner solution involve monotonic sequence of employment rates
  (iii) If \( z \geq S \) the sequence of employment rate is increasing and bounded from above by 1
  (iii) If \( z < S \) the sequence of employment rate is decreasing and bounded from below by 0

• To check that there is no interior solution simply not that a fix point of (33) is the solution to:
  \[ \tilde{u}^{my}(E(x), x) = x \]
  which is \( x = -\frac{\eta_{u}}{\eta_{l}+1} < 0 \), hence a steady state is necessarily a corner solution.

  (i) Assume that \( \tilde{u}^{a}_{t+1} = 1 \) then we should show that \( \frac{\partial Y(l_{t+1}x)}{\partial l_{t}} > 0 \) for \( \tilde{u}_{t} \in [0, 1] \). Namely, when the current state of the economy is such that the proportion of educated insiders is compatible with full employment, then insiders expecting full employment will also choose full employment. Indeed one can check that:
  \[ \tilde{u}^{my}_{t}(x, 1) = \frac{\Delta \eta}{S-1} x + \frac{\eta_{u} (z-S) - S}{(S-1) z} = 1 \iff (z-S)(z+1+\eta_{u}) \geq 0 \]
  hence \( \tilde{u}^{my}_{t} = 1 \) is a rational expectation equilibrium if \( z \geq S \)

  (ii) Assume that \( \tilde{u}^{a}_{t+1} = 0 \) then \( \tilde{u}^{my}_{t} = 0 \) is a steady state equilibrium if \( \tilde{u}^{my}_{t}(0, 0) \leq 0 \) which is true if \( z < S \). Rather if \( z \geq S \) then the policy \( \tilde{u}^{my}_{t} = 0 \) is never a steady state rational expectation with myopic insiders.

• Rational expectations are monotonic. This follows from the observation made in the text in Figures 5 and Figures 6. Namely if a current insider find it profitable to increase minimum wages then it is still more profitable for the next period insiders to keep increasing minimum wages. While if it were profitable for insiders to lower minimum wages it still more profitable for the next period insiders to keep doing so. If expectations are rational then they should verify that if \( l_{t} > l^{a}_{t+1} \) then \( l_{t+1} > l^{a}_{t+2} = l_{t+2} > ...l_{t+j} = l_{t+j+1} \) and if \( l_{t} < l^{a}_{t+1} \) then \( l_{t+1} < l^{a}_{t+2} = l_{t+2} < ...l_{t+j} \).

• Assume \( z < S \) and that \( l^{a}_{t+1} < \tilde{u}^{my}_{t} \) then with rational expectations \( E_{t+2} < E_{t+1} \) and by backward induction \( E_{t+1} < E_{t} \), which is compatible with (33), hence by induction we can find a sequence of interior solution starting from \( l_{t} \) such that: \( l_{t} > l^{a}_{t+1} = l^{a}_{t+2} = l_{t+2} > ...l_{t+j} = l_{t+j+1} \). All rational expectations equilibrium, this sequence is decreasing and bounded hence it converges to \( \tilde{u}^{my}_{t} = 0 \) which have been shown is a steady state rational expectation equilibrium if \( z < S \).

  Assume \( z < S \) and that \( l^{a}_{t+1} > \tilde{u}^{my}_{t} \) then with rational expectation \( E_{t+2} > E_{t+1} \) and by backward induction \( E_{t+1} > E_{t} \), inspecting (33) one has that \( l^{a}_{t+1} = \frac{S}{l_{t}+\eta_{u}+E_{t}+\Delta_{u}} \). In this case and for all \( \tilde{u}^{my}_{t} \) such that \( \tilde{u}^{my}_{t} < l^{a}_{t+1} \) is true, the optimal choice is \( \tilde{u}^{my}_{t} = 0 \) (since the objective function is decreasing in \( [0, l^{a}_{t+1}] \)).

  Hence if \( z < S \) the economy with myopic has a unique steady state equilibrium with \( \tilde{u} = 0 \).
• Assume now that \( z > S \) and that \( \ln_{t+1} \leq \tilde{I}_t \) then with rational expectation \( E_{t+2} < E_t \) and by backward induction \( E_{t+1} < E_t \), inspecting \(^{33}\) one has that \( \frac{S}{\tilde{I}_t + \eta_e + E_t \Delta \eta} < \frac{z}{\tilde{I}_t + \eta_e + E_t \Delta \eta} \) in this case and for the set of policies such that \( \ln_{t+1} < \ln_{t}^{my} \) the optimal choice is \( \ln_{t}^{my} = 1 \), since the objective function is increasing in \([\ln_{t+1}^{s}, 1]\) for \( z > S \).

- Assume rather that \( \ln_{t+1} > \tilde{I}_t \) then with rational expectations \( E_{t+2} > E_t \) and by backward induction \( E_{t+1} > E_t \) \( \Leftrightarrow \tilde{I}_t > \tilde{I}_{t-1} \) clearly there exists an interior \( \tilde{I}_t \) compatible with the FOC \(^{33}\) by induction one can construct an increasing rational expectation sequence \( l_t < \ln_{t+1} = l_{t+1} < \ln_{t+2} = l_{t+2} < \ln_{t+3} = l_{t+3} < \ldots < \ln_{t+j} = 1 = l_{t+j} = l_{t+j+1} \), which is bounded by \( \tilde{I}_t = 1 \) hence starting from \( \ln_{t+1} > \tilde{I}_t \) the economy converges to \( \tilde{I}_t = 1 \).

And we have shown that \( \ln_{t}^{my} = 1 \) is a steady state rational expectation equilibrium.

To sum-up

• With \( z > S \)

The unique steady state rational expectation equilibrium with myopic insider display full employment.

• With \( z < S \),

The unique steady state rational expectation equilibrium with myopic insider display full unemployment for youths.

**Proof. Proposition (4).** Assume \( (S - 1) < z \leq (S - 1) c \) and that \( \eta_e < (S - 1) (S + 1) \)

The capital stock evolves according to the following law of motion:

\[
K_{t+1}(E_t) = \frac{\beta}{1+\beta} A(1-\alpha) \left( \frac{K_t}{H(\tilde{L}(E_t); E_t)} \right)^\alpha * L_t^2(E_t)
\]

We now that the policy variable converges to two possible steady state equilibrium, one with full employment, \( \tilde{L}(E_t) = \tilde{L}(E_{t+1}) = 1 \) and \( E_t = E_{t+1} = x \), and the other with youths being partly unemployed, \( \tilde{L}(E_t) = \tilde{L}(E_{t+1}) = \frac{1-b}{a} \) and \( E_t = E_{t+1} = \frac{1-b}{a} \). Each steady state policy corresponds to different steady state equilibrium growth path for the capital stock:

\[
K_{t+1}^{fe} = \frac{\beta}{1+\beta} A(1-\alpha) \left( \frac{K_t^{fe}}{H(1)} \right)^\alpha * L_t^2(x)
\]

\[
K_{t+1}^{yu} = \frac{\beta}{1+\beta} A(1-\alpha) \left( \frac{K_t^{yu}}{H(\frac{1-b}{a})} \right)^\alpha * L_t^2(\frac{1-b}{a})
\]

From these laws of motion it is straightforward to deduce the corresponding steady state aggregate capital, capital labor ratio, and aggregate output.

• \( (K_{t}^{fe})^{1-\alpha} = \left( \frac{1}{1+\eta_e x + \Delta \eta x} \right)^\alpha (\eta_e + \Delta \eta x) > \left( \frac{1}{\eta_e x + \Delta \eta x} \right)^\alpha (\eta_e + \Delta \eta x) \) to compare these expressions let’s define the following useful function:

\[
\ln(f(u)) = -\alpha \ln \left( \frac{u}{x} + \eta_e + \Delta \eta u \right) + \ln (\eta_e + \Delta \eta u)
\]
and \( u \in D = [\frac{1-b}{a}, x] \), one see that \( f(\frac{1-b}{a}) = (K_{gu}^*)^{1-\alpha} \) and \( f(x) = (K_{fe}^*)^{1-\alpha} \) to prove that \( K_{fe}^* > K_{gu}^* \) we shall show that the function is increasing over the domain \( D \), that is

\[
\alpha \frac{1/x + \Delta \eta}{\eta_e + \Delta \eta u} < \frac{\Delta \eta}{\eta_e + \Delta \eta u}
\]

a sufficient condition for the previous inequality to hold is that \( \alpha < (1 - \alpha) x\Delta \eta \equiv (1 - \alpha) z \) since we assumed that \( z \in ((S - 1), c(S - 1)) \) it suffices to show that is true for \( S - 1 \), \( \frac{\alpha}{1-\alpha} < S - 1 \) replacing \( S \) by its value this is equivalent to \( 1 < \frac{\alpha(1+\alpha\beta)}{\beta} \) a sufficient condition\(^1\) is that \( \alpha > \beta \) which is indeed a sufficient condition for \( S > 1 \).

- We show next that steady state labor productivity that is the capital labor ratio (productivity per hour worked ) is higher in the unemployment equilibrium:

\[
k_{yu}^* = \frac{K_{yu}^*}{H(\frac{1-b}{ax})} > k_{fe}^* = \frac{K_{fe}^*}{H(\frac{1-b}{ax})} \Leftarrow L^2(\frac{1-b}{a}x) > L^2(x)
\]

\[
\Leftrightarrow \frac{1 + \eta_e + \Delta \eta x}{\eta_e + \Delta \eta x} > \frac{1-b}{ax} + \frac{\Delta \eta 1-b}{\eta_e + \Delta \eta a}
\]

We use the same trick as before and define the function \( g(u) = \frac{\eta_e + \Delta \eta u}{\eta_e + \Delta \eta u} \) on the domain \( D = [\frac{1-b}{a}, x] \) and by noting that \( g(x) \) is equal to the \( LHS \) of the inequality, we have to show that \( g'(x) > 0 \) for the inequality to hold, indeed we can check that \( g'(u) = \frac{\eta_e/x}{(\eta_e + \Delta \eta u)^2} > 0 \). This proves that \( k_{yu}^* > k_{fe}^* \).

- Proving \( Y_{fe}^* > Y_{yu}^* \) is straightforward owing to the Cobb Douglas production function,

\[
Y_{fe}^* > Y_{yu}^* \Leftrightarrow L_{fe} * \left( \frac{K_{fe}^*}{L_{fe}} \right)^\alpha > L_{yu} * \left( \frac{K_{yu}^*}{L_{yu}} \right)^\alpha
\]

with more capital and more labor clearly \( Y_{fe}^* > Y_{yu}^* \).

\[\text{Comparative static with respect to } z \text{ and } S\]

- High unemployment equilibrium

\[\text{Change in } z\]

In the high unemployment equilibrium the stock of educated insiders labor is \( \frac{1-b}{a} \) and the equilibrium youth employment rate is:

\[
\frac{1-b}{ax} = \frac{(S - 1)(z + 1) - \eta_u (z - (S - 1))}{(z + 1) z}
\]

(34)

taking the derivative with respect to \( z \):

\[
\frac{-z^2 ((S - 1) - \eta_u) - (2z + 1)(S - 1)(1 + \eta_u)}{(z + 1) z} < 0
\]

\[\text{Indeed this is also a sufficient condition for the economy to be dynamically efficient if capital market were perfect.}\]
Hence a marginal increase in $z$ increases equilibrium minimum wage.

*Change in $S$*

Observing (34) it is clear that as $S$ increases the equilibrium preferred youth employment rate rises, consequently the equilibrium human capital human capital increases.

- High employment equilibrium

  *Change in $z$*

  In the high employment the equilibrium policy is simply $\bar{l} = 1$ and the enrollment rate is $x$, which is an increasing function of $z$.

  *Change in $S$*

  Observing (34) it is clear that as $S$ increases the equilibrium preferred youth employment rate rises, consequently the equilibrium human capital human capital increases.

**Effect on productivity**

$$k = \left(\frac{\eta + \Delta\eta x}{1 + \eta + \Delta\eta x}\right)^{\frac{1}{1-n}}$$

Clearly as $z$ increases $x$ increases and the productivity increases.
B Wage policy in an endogenous growth model

Suppose that $A$ rather than being a parameter is endogenous and given by:

$$A_t = AK_t^{1-\alpha}$$ (35)

Thus the state of the technology rather than being fix evolves as a function of the aggregate level of capital. Equation (35) can be motivated by the assumption of production externalities. Individuals firms behave competitively and maximise profits taking $A_t$ as given. This leads to an aggregate production function featuring constant marginal return to capital:

$$Y = AK_t H_t^{1-\alpha}$$

As in Romer (1989) production is linear in capital, it is indeed and $AK$ type model of endogenous growth. In this case the equilibrium factor prices are:

$$W_t = (1 - \alpha) A K_t / H_t^\alpha$$

and

$$R_t = \alpha A H_t^{1-\alpha}$$

The capital market equilibrium condition implies:

$$L_t^i \geq \frac{\beta}{1 + \beta} (1 - \alpha) A K_t / H_t^{\alpha} = K_{t+1}$$

Hence a growth factor during the transition to the steady state growth is:

$$\frac{Y_{t+1}}{Y_t} = L_t^i \beta \frac{(1 - \alpha)A H_t^{1-\alpha}}{H_t}$$ (36)

Hence the economy with the higher employment rate has also the higher growth rate.

The insider’s objective function is:

$$V^2(\tilde{l}_t; E_t) = -\hat{S} \ln H_t(\tilde{l}_t, E_t) + \ln H_{t+1}(\tilde{l}_{t+1}(\tilde{l}_t), E(\tilde{l}_t))$$

with $\tilde{l}_t, \tilde{l}_{t+1} \in [0, 1]$ and $E_{t+1} = \bar{x} \tilde{l}_t$

The problem is the same as the one solved in the main text with $\hat{S} = \alpha(1+\beta) / \beta(1-\alpha) > S$, replacing $S$.

Since $\hat{S} = \alpha(1+\beta) / \beta(1-\alpha) > S$, a necessary condition for high employment equilibrium is $S < \hat{S} < z$, hence the case for high unemployment equilibrium is more likely with capital externality in production. We can also show that steady state growth rate is higher in the high employment equilibrium:

$$\frac{Y_{t+1}}{Y_t} = L_t^i \beta (1 - \alpha) A H_t^{1-\alpha}$$

at the high employment steady state $H_{t+1} = H_t = H(1)$ and $L_t^i = L^i(\bar{x})$, while in the low employment steady state $H_{t+1} = H_t = H(1-b/\alpha)$ and $L_t^i = L^i(1-b/\alpha)$. The steady state growth rate in the high employment equilibrium is:

$$\left(\frac{Y_{t+1}}{Y_t}\right)^c = L^i(\bar{x}) \frac{\beta (1 - \alpha) A}{1 + \beta (H(1))^{\alpha}}$$
while in the low employment equilibrium it is:

\[(Y_{t+1}/Y_t)^u = L' \left( \frac{1-b}{a} \right) \frac{\beta}{1+\beta \left( H \left( \frac{1-b}{ax} \right) \right)}\]

\[(Y_{t+1}/Y_t)^c > (Y_{t+1}/Y_t)^u \Leftrightarrow \frac{\eta_u + \Delta \eta x}{(1 + \eta_x + \Delta \eta x)^a} > \frac{\eta_u + \Delta \eta \frac{1-b}{a}}{(1-b/a + \eta_u + \Delta \eta \frac{1-b}{a})^a}\]

which is the case provided that \(z > \frac{\alpha}{1-\alpha}\). The parameter range for multiple steady state implies that \(z > \hat{S}\) and since \(\hat{S} > \frac{\alpha}{1-\alpha}\) one has \(z > \frac{\alpha}{1-\alpha}\). We conclude that the steady state growth rate is higher in the high employment equilibrium.
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