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Dynamically consistent CEU preferences

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Abstract

We give an axiomatic foundation to the updating rule proposed by [Sarin, R. and Wakker, P. P. (1998). Revealed likelihood and knightian uncertainty. Journal of Risk and Uncertainty 16(3):223-250.] for CEU preferences. This rule is dynamically consistent but non-consequentialist, since forgone consequences are relevant for conditioning. Whereas it does not work universally, but only when counterfactuals outcomes are better and/or worse than the ones resulting on the conditioning event, the rule has many interesting features, since it is able to describe Ellsberg-type preferences together with a recursive structure of the criterion.

Keywords: Choquet Expected Utility; Capacities; Dynamic consistency; Updating
JEL classification numbers: D 81, D 83

1 Introduction

Ambiguity is defined as a context in which probabilistic information about events is not given to the decision maker. In decision theory under ambiguity, typical violations of the standard model have led to the development of several generalizations, known as non-expected utility criteria. In this paper, we will focus on one of the most popular approaches, namely the Choquet Expected Utility (CEU) model. This model, firstly axiomatized by Gilboa (1987) and Schmeidler (1989), assumes that the decision maker's beliefs are represented by a non-necessarily additive probability, or capacity. It has been successfully applied to various economic situations, notably in applications of decision theory (finance, insurance...). However, a wide range of economic problems involves sequential resolution of the uncertainty, and the use of CEU model in this context is a major concern.

In such situations, we have to integrate new information in the decision process. How to perform it is often specified by a set of axioms applied on individual preferences (see, for instance, Gilboa and Schmeidler, 1993, and Eichberger et al., 2007). Two important axioms are consequentialism and dynamic consistency. The former implies that conditional preferences only depends on the conditioning event, while the latter links unconditional and conditional preferences. These two assumptions together are incompatible with non-additive beliefs (e.g. Ghirardato, 2002, and Lapied and Toquebeuf, 2010): under consequentialism, CEU preferences are dynamically consistent if and only if beliefs are additive

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and updated according to Bayes rule. Consequently, axiomatic works cited above keep consequentialism but weaken dynamic consistency[1]. Further, even if consequentialism is not assumed, dynamic consistency is not, in general, satisfied by CEU preferences (see Epstein and Le Breton, 1993, and Eichberger and Kelsey, 1996).

Sarin and Wakker (1998a) have proposed an updating rule for CEU preferences. Since they argue that it is dynamically consistent but non-consequentialist, in the sense that counterfactual outcomes may be relevant for conditioning, it has to be studied in an axiomatic framework. Dropping consequentialism, when it is possible, may have interesting implications for behavior under uncertainty, as illustrated by Machina (1989). In this paper, we show that CEU preferences satisfy a restricted version of dynamic consistency - since it only holds when the conditioning event is f-convex, if and only if they are updated according to the rule proposed by Sarin and Wakker. Nevertheless, conditional preferences are only defined for f-convex events: the price to pay for relaxing consequentialism. Stated otherwise, the largest set of acts on which CEU preferences exhibit a recursive structure consists in those acts that are comonotonic with their conditional expectations.

The paper is organized as follows. The next section presents our set-up and axioms. In section 3, we report the main result and discuss some of its implications.

2 Set-up and axioms

2.1 Set-up

We consider a *state space* S containing elements denoted by s. An event is a subset of S and for all $A \subset S$ we note A^c the event $S - A$. S is endowed with a sigma-algebra noted Σ so that (S, Σ) is a measurable space. The set of *outcomes* is X and it is assumed to be a connected and separable topological space. The set of simple acts $f : S \to X$, that are Σ-measurable functions taking only finite values, is denoted by $\mathcal{A}$, and f_E^g refers to the act h yielding $h(s) = f(s)$ when $s \in E$ and $h(s) = g(s)$ when $s \in E^c$.

A decision maker (DM for short) is characterized by a class of preference relations $\{\succcurlyeq_E\}_{E \in \Sigma}$ on $\mathcal{A}$. For all $E \in \Sigma$, the conditional preference $\succcurlyeq_E$ compares acts when she is informed that the right state is in E. When $E = S$, $\succcurlyeq_S$ is noted $\succcurlyeq$ and denotes the unconditional preference. Abusing notations, we consider that X is the set of constant acts $f(.)$ such that $f(s) = x$ for all $s \in S$. Then the preference relation $\succcurlyeq$ on $\mathcal{A}$ induces a preference relation on X, also denoted by $\succcurlyeq$. We assume that the DM's tastes are stable, in the sense that for all $E \in \Sigma$, $\succcurlyeq_E \equiv \succcurlyeq$ on X. Then $\succcurlyeq$ on X is represented by a continuous and strictly increasing utility function $u : X \to R$ that does not change with information arrivals. In other words, $u(.)$ is *state-independent*. Throughout we shall assume the following structural assumption on X :

Assumption 1 (Non-triviality)**.** *There exist a best and a worst outcome, i.e.* $\exists x^*, x_* \in X$ *such that* $x^* \succ x_*$ *and* $x^* \succcurlyeq x \succcurlyeq x_*$ *for all* $x \in X$*.*

The present work assumes that each preference relation $\succcurlyeq_E$ is represented by a Choquet Expected Utility (CEU) functional[2] noted $I_E[u(.)]$. In this model, the DM's beliefs are

[1] Another solution consists to specify information sets by a given and fixed filtration, see for instance Sarin and Wakker (1998b), Eichberger et al. (2005), and Dominiak and Lefort (2011).

[2] Equivalently, one could assume that each preference relation satisfy a set of axioms necessary and sufficient to the existence of a CEU representation. See, among others, Gilboa (1987) and Wakker (1989) (for the finite case).

represented by a normalized and monotonic set function $\nu_E : \Sigma \to \mathbb{R}$, called *Choquet capacity*, such that (i) $\nu_E(E) = 1$ and $\nu_E(E^c) = 0$ and (ii) for all $A, B \in \Sigma, A \subseteq B \Rightarrow \nu_E(A) \leq \nu_E(B)$. Further, when $E = S$, $\nu_S(.)$ is simply noted $\nu(.)$. It is said to be convex (concave) if for all $A, B \in \Sigma$, we have $\nu(A) + \nu(B) \leq (\geq)\nu(A \cup B) + \nu(A \cap B)$, and it is a probability if $\leq$ ($\geq$) is replaced by $=$. In order to define the Choquet integral, we associate to any act $f \in \mathcal{A}$ the coarsest finite partition over S, $\mathcal{P}_f = \{A_1, ..., A_n\}$, to which f is measurable and ordered. Therefore, if $s, s' \in A_i$, for any $i = 1, ..., n$, then $f(s) \sim f(s')$ and for $i, j = 1, ..., n, i < j$, $s \in A_i$ and $s' \in A_j$ imply $f(s') \succcurlyeq f(s)$. Hence the CEU of act $f \equiv (x_i, A_i)_{i=1}^n$ with respect to $\nu(.)$, noted $I[u(f)]$, can be written as

$$\sum_{i=1}^{n-1} u(x_i) \cdot [\nu(A_i \cup ... \cup A_n) - \nu(A_{i+1} \cup ... \cup A_n)] + u(x_n)\nu(A_n)$$

We denote by $\Sigma(f) = \{A \subset \mathcal{P}_f\}$ the algebra generated by f. Then $p^f : \Sigma(f) \to [0, 1]$ is the probability (or "decision weight" in the terminology of Sarin and Wakker, 1998a) used to value f, such that $p^f(A_i) = \nu(A_i \cup ... \cup A_n) - \nu(A_{i+1} \cup ... \cup A_n)$ for $i < n$ and $p^f(A_n) = \nu(A_n)$. Two acts f and g are said to be *comonotonic* if there are no $s, s' \in S$ such that $f(s) \succ f(s')$ and $g(s') \succ g(s)$. The following proposition is straightforward:

Proposition 1. *For two comonotonic acts $f, g \in \mathcal{A}$, if $\Sigma(f) \cap \Sigma(g) \neq \emptyset$ then $\forall A \in \Sigma(f) \cap \Sigma(g)$, $p^f(A) = p^g(A)$.*

This property of the CEU model is closed to the "tail-separability" of Machina (2009). The following definition, proposed by Eichberger and Kelsey (1996), will be useful throughout.

Definition 1 (Non-null event)**.** *An event $E \in \Sigma$ is said to be non-null if $\nu(B \cup E) > \nu(B)$ for any $B \in \Sigma$ such that $B \cap E = \emptyset$.*

In the sequel, we will assume that the conditioning event E is non-null, otherwise $I_E[u(.)]$ will be not defined.

Definition 2 (f-convexity)**.** *An event E is f-convex[3] if for all $s', s'' \in E$ and $s \in S$, if $f(s') \succcurlyeq f(s) \succcurlyeq f(s'')$ then $s \in E$.*

This property has firstly been proposed by Gilboa (1987). For instance, for $h \equiv (x^*, A; x_*, A^c)$ and $f \equiv x^h_E$, E is straightforwardly f-convex. In words, we say that event E is $f-$convex if consequences of f outside of E are unanimously preferred and/or un-preferred to consequences yielding by f on E. In the context of updating of CEU preferences, f-convexity was implicit in Chateauneuf et al. (2001) and made explicit by Koida (2010).

2.2 Dynamic consistency and consequentialism

A common definition of dynamic consistency[4] requires that an act is unconditionally weakly preferred to an other if and only if it is also weakly preferred conditionally to the event on which these acts differ. Such a requirement is clearly too strong in the Choquet framework and it is well known that strictly non-additive CEU preferences cannot satisfy this axiom (see Eichberger and Kelsey 1996). Hence we restrict it to f-convex conditioning events.

[3] Sarin and Wakker (1998a) called such an event "connected".

[4] See, for instance, Epstein and Le Breton (1993), Sarin and Wakker (1998b), Ghirardato (2002) and Dominiak and Lefort (2011).

Axiom 1 (f-convex Dynamic Consistency). *For all $f, g \in \mathcal{A}$ and any non-null event $E \in \Sigma$ such that E is both f-convex and g_E^f-convex, $f \succcurlyeq g_E^f$ if and only if $f \succcurlyeq_E g_E^f$.*

Most of approaches of updating of non-expected utility preferences assume the following axiom:

Axiom 2 (Consequentialism). *For all $f, g \in \mathcal{A}$ and $E \in \Sigma$, $f \sim_E f_E^g$.*

Consequentialism makes it easier to define conditional beliefs when new information is gathered. It means that conditional preferences only depends on the conditioning event. Therefore, it can be divided in two parts:

(i) Irrelevance of forgone consequences (axiom 2 above);

(ii) History-independence of conditional preferences: the DM does not look backward to make her later choices.

Part (ii) is implicit in our set-up. On the opposite, within the multiple prior framework, Hanany and Klibanoff (2007,2009) keep part (i) but drop (ii): in their approach, conditioning depends on the initially chosen act.

Unfortunately, since we do not assume consequentialism, each conditional CEU $I_E[u(.)]$ is only defined when E is f-convex. Nevertheless, dropping consequentialism may have sense for non-expected utility maximizers, as illustrated by Machina (1989).

3 Results and discussion

3.1 The main result

We state that f-convex dynamic consistency holds if and only if conditional decision weights are given by the updating rule proposed by Sarin and Wakker (1998a). In this case, the value of any $f \equiv (x_i, A_i)_{i=1}^n$ such that $x_n \succcurlyeq ... \succcurlyeq x_1$, conditional to any non-null and f-convex $E \in \Sigma$, is given by:

$$I_E[u(f)] = \sum_{i=1}^{n} u(x_i) \frac{p^f(A_i \cap E)}{p^f(E)} \tag{1}$$

Then the SW rule consists to apply the Bayes updating rule for probabilities to $p^f(.)$:

$$p_E^f(A) = \frac{p^f(A \cap E)}{p^f(E)} \tag{2}$$

Further, it contains an implicit definition of the conditional capacity $\nu_E^f(.)$:

$$\nu_E^f(A) = \frac{\nu((A \cap E) \cup D_f(E)) - \nu(D_f(E))}{\nu(E \cup D_f(E)) - \nu(D_f(E))} \tag{3}$$

where $D_f(E) = \{s \in E^c | \forall s' \in E, f(s) \succcurlyeq f(s')\}$, hence it is named "dominating event" by Sarin and Wakker (1998a). Then an alternative way of writing the SW rule consists to consider the capacity $\nu(.)$ rather than the decision weight $p^f(.)$.

Proposition 2. *For any $f \in \mathcal{A}$, let $E \in \Sigma$ be f-convex and non-null. Then,*

$$\int_S u(f) d\nu_E^f = \sum_{i=1}^{n} u(x_i) \frac{p^f(A_i \cap E)}{p^f(E)} \tag{4}$$

where $\nu_E^f(.)$ is given by eq. (3).

Proof W.l.o.g., let $f = (x_i, A_i)_{i=1}^n$ such that $x_n \succcurlyeq ... \succcurlyeq x_1$. Let $A^c \cap E^c = \cup_{i=1}^k A_i$, $E = \cup_{i=k+1}^l A_i$ and $A \cap E^c = \cup_{i=l+1}^n A_i$. We have:

$$\int_S u \circ f d\nu_E^f = \sum_{i=1}^n u(x_i)[\nu_E^f(A_i \cup ... \cup A_n) - \nu_E^f(A_{i+1} \cup ... \cup A_n)] \tag{5}$$

where $\nu_E^f(A_i \cup ... \cup A_n) - \nu_E(A_{i+1} \cup ... \cup A_n)$ is equal to

$$\frac{\nu(A_i \cup ... \cup A_l \cup D_f(E)) - \nu(A_{i+1} \cup ... \cup A_l \cup D_f(E))}{\nu(E \cup D_f(E)) - \nu(D_f(E))} \tag{6}$$

where $D_f(E) = A \cap E^c$ and the denominator is not equal to zero since E is non-null. Since

$$p^f(E) = \sum_{i=k+1}^l [\nu(A_i \cup ... \cup A_n) - \nu(A_{i+1} \cup ... \cup A_n)] \tag{7}$$

$$= \nu(E \cup D_f(E)) - \nu(D_f(E)) \tag{8}$$

and

$$p^f(A_i \cap E) = \nu(A_i \cup ... \cup A_l \cup D_f(E)) - \nu(A_{i+1} \cup ... \cup A_l \cup D_f(E)) \tag{9}$$

for all $i = k+1, ..., l$, eq. (5) is equivalent to eq. (4) with $\nu_E^f(.)$ defined by eq. (3). □

Therefore, the Choquet expectation of $u \circ f$ w.r.t. $\nu_E^f(.)$ and its additive expectation w.r.t. $p_E^f(.)$ coincide. The updating rule of eq. (3) has been separately studied by Young (1998), who named it the "Generalized Bayes rule", since it generalizes the Gilboa and Schmeidler $f-$Bayesian approach (see subsection 3.2 below). The following proposition lists some elementary properties of $\nu_E^f(.)$ and, together with proposition 2, it ensures that $I_E[u(f)]$ defined by eq. (1) is a conditional CEU.

Proposition 3. *Let $f \in \mathcal{A}$, let $E \in \Sigma$ be non-null and f-convex and let $\nu_E^f(.)$ be given by eq. (3). Then the following statements hold:*

(i) If $\nu(.)$ is convex (concave) then $\nu_E^f(.)$ is convex (concave), too;

(ii) If $\nu(.)$ is additive, then $\nu_E^f(.)$ is additive, too, and it is given by the Bayes updating rule for probabilities;

(iii) $\nu_E^f(S) = \nu_E^f(E) = 1$, $\nu_E^f(\emptyset) = \nu_E^f(E^c) = 0$, and $\forall A, B \in \Sigma, A \subseteq B \Rightarrow \nu_E^f(A) \leq \nu_E^f(B)$;

Proof We just prove statement (i). Other statements are straightforward. If $\nu(.)$ is convex (concave), then, given $A_1, A_2 \in \Sigma$,

$$\begin{aligned}&\nu(A_1 \cap E) \cup D_f(E)) + \nu(A_2 \cap E) \cup D_f(E)) \\ &\leq (\geq)\nu[((A_1 \cap E) \cup D_f(E)) \cup ((A_2 \cap E) \cup D_f(E))] \\ &+ \nu[((A_1 \cap E) \cup D_f(E)) \cap ((A_2 \cap E) \cup D_f(E))]\end{aligned}$$

hence

$$\begin{aligned}&\nu((A_1 \cap E) \cup D_f(E)) + \nu((A_2 \cap E) \cup D_f(E)) \\ &\leq (\geq)\nu[((A_1 \cup A_2) \cap E) \cup D_f(E)] + \nu[((A_1 \cap A_2) \cap E) \cup D_f(E)]\end{aligned}$$

Therefore,

$$\nu_E^f(A_1) + \nu_E^f(A_2) \leq (\geq) \nu_E^f(A_1 \cup A_2) + \nu_E^f(A_1 \cap A_2)$$

□

The reader is referred to Young (1998) for a wider study of (statistical) properties of the SW rule. Now we state our main result, that characterizes the conditional CEU representation of $\succcurlyeq_E$.

Theorem 1. *Let $\{\succcurlyeq_E\}_{E\in\Sigma}$ be a class of preference relations on $\mathcal{A}$ represented by $I_E[u(.)]$ for all $E \in \Sigma$. Then the following statements are equivalent :*

(i) $\{\succcurlyeq_E\}_{E\in\Sigma}$ satisfy f-convex dynamic consistency;

(ii) For any $f \in \mathcal{A}$ and any non-null f-convex $E \in \Sigma$, $I_E[u(f)]$ is given by eq. (1).

Lemma 1. *Let $E \in \Sigma$ be non-null and f-convex. Then there exists a unique $x \in X$ satisfying*

$$u(x) = \int_S \frac{u(f)1_E}{p^f(E)} dp^f \tag{10}$$

where 1_E is the characteristic function of E.

Proof Since X is a connected and separable topological space, continuity of the CEU representation and assumption 1 imply that there exists $x \in X$ such that $f \sim x_E^f$. Further, x is unique by monotonicity of $\succcurlyeq$. Moreover, $f(s) \succcurlyeq x \succcurlyeq f(s')$ for any $s \in D_f(E)$ and any $s' \in \bar{D}_f(E)$, hence x_E^f is comonotonic with f. In addition, $\Sigma(x_E^f) \subset \Sigma(f)$. Therefore, by proposition 1, we have $I(x_E^f) = \int_S u(x_E^f) dp^f$, hence $I[u(f)] = I[u(x_E^f)]$ if and only if $I[u(f)1_E] = I[u(x_E^f)1_E]$ if and only if $u(x) = I[u(f)1_E]/p^f(E)$. Finally, E is assumed to be non-null, hence $p^f(E) > 0$ thus eq. (10) is well defined. □

Proof of the theorem **Part A** $(i) \Rightarrow (ii)$. Let $E \in \Sigma$ be f-convex. Then, by lemma 1, there exists $x \in X$ such that eq. (10) holds. Further, under axiom (1), we have $f \sim x_E^f$ if and only if $u(x) = I_E[u(f)]$, hence $I_E[u(f)]$ is defined by eq. (1), as claimed.

Part B $(ii) \Rightarrow (i)$. By lemma 1, for all $f, g \in \mathcal{A}$ such that E is both f-convex and g_E^f-convex, we have $f \succcurlyeq g$ if and only if $x_E f \succcurlyeq y_E f$, where, under the SW rule, $u(x) = I_E[u(f)]$ and $u(y) = I_E[u(g_E^f)]$. By monotonicity of $\succcurlyeq$, $x_E^f \succcurlyeq y_E^f$ implies $x \succcurlyeq y$, that is equivalent to $f \succcurlyeq_E g_E^f$. □

It is worth noting that, as in the Bayesian approach, preferences are dynamically consistent if and only if they are updated according to the Bayes rule. The SW rule is, by nature, restricted to conditioning events sharing the property of f-convexity. Indeed, applying the SW rule on a larger set of conditioning events entails that $\nu_E^f(.)$ in eq. (3) may be non-normalized or non-monotonic w.r.t. set inclusion. Therefore, if $\nu(.)$ is strictly non-additive, the set of $f-$convex events is the largest set on which $\succcurlyeq$ admit a dynamically consistent updating.

3.2 Gilboa and Schmeidler and the $f-$Bayesian approach

The SW rule axiomatized in the present work rests on the $f-$Bayesian approach initiated by Gilboa and Schmeidler (1993). Updating rules sharing this property are, notably, the

Bayes rule and the Dempster-Shafer rule. If we consider eq. (3), we observe that the former is obtained when $E^c = \bar{D}_f(E)$:

$$\nu_E^f(A) = \frac{\nu(A \cap E)}{\nu(E)} \tag{11}$$

whereas the latter is used when $E^c = D_f(E)$:

$$\nu_E^f(A) = \frac{\nu((A \cap E) \cup E^c) - \nu(E^c)}{1 - \nu(E^c)} \tag{12}$$

Then it is possible to adopt different treatment of ambiguity, depending on the nature of information. Good news involve the DM uses the Bayes update whereas bad news imply the use of the Dempster-Shafer update. Alternating these two updating rules has already been made by Chateauneuf et al. (2001), with more restrictive assumptions.

3.3 The Ellsberg's paradox

Contrarily to other recursive models of choice under ambiguity, the one we propose is able to describe a dynamic version of the Ellsberg's paradox, similar to Epstein and Schneider (2003) example 4-1[5].

A CEU decision maker is facing an urn containing 30 red balls and 60 blue or green balls. At time 1, a ball is drawn and the decision maker knows whether this ball is green or not. At time 2, the color of this ball is fully revealed to the decision maker. Then the state space S is partitioned by events R, B and G, that have obvious signification, and conditional preferences are $\{\succcurlyeq_{R\cup B}, \succcurlyeq_G\}$. Let $E = R \cup B$. Several bets, that are $\{R, B, G\}$-measurable maps from S to $X = \{0, 1\}$, are proposed to the DM. Let $f \equiv (1, 0, 0), g \equiv (0, 1, 0), f' \equiv f_E^1$ and $g' \equiv g_E^1$. Ellsberg-type preferences are $f \succ g$ and $g' \succ f'$, hence a possible capacity is $\nu(B) = \nu(G) = 1/6$ and $\nu(R \cup B) = \nu(R \cup G) = 1/2$. For simplicity, assume that the utility $u(.)$ is the identity. Note that the conditioning event E is both f_E^x-convex and g_E^x-convex, for any $x \in X$. Then, by theorem 1, f-convex dynamic consistency holds if and only if conditional Choquet preferences are given by:

$$I_E(f) = \frac{2}{3} > \frac{1}{3} = I_E(g) \tag{13}$$

and

$$I_E(f') = \frac{2}{5} < \frac{3}{5} = I_E(g') \tag{14}$$

according to ex-ante preferences. In eq. (13), conditional capacities are given by the Bayes rule, whereas the DM uses the Dempster-Shafer update in eq. (14). Therefore, one can state the following observation:

Observation 1. *Consequentialism is violated by eq. (13) and (14) since $f \succ_E g$ and $g_E^1 \succ_E f_E^1$.*

This example makes it particularly clear that ambiguity aversion is not, by itself, contradictory with dynamic consistency if consequentialism is dropped.

[5] A similar extension of this famous paradox is experimented by Dominiak, Dursch and Lefort (2009).

3.4 The law of iterated expectations

In this subsection, we turn our attention on cases with a number $m \geq 2$ of conditioning events in order to present relations among the law of iterated expectations, dynamic consistency, and the SW update rule. Then we consider a class of CEU preferences $\{\succcurlyeq_j\}_{j=0,...,m}$ where $\succcurlyeq_j$ designates the conditional preference to event E_j and the unconditional preference when $j = 0$, such that $E_0 = S$. Similarly, $I_j[u(.)]$ is the CEU functional w.r.t. capacity $\nu_j(.)$, conditional on event E_j, and $I_0[u(.)]$ is the CEU representation of $\succcurlyeq_0$.

Let $f_j \in X$ be the conditional certainty equivalent of act f on event E_j, such that $f_j := u^{-1}[I_j(u(f))]$. Then $\tilde{f} \equiv [f_j, E_j]_{j=1}^m$ refers to the *conditional certainty equivalent act* of f. An important feature of the SW rule is that it allows the *law of iterated expectations* to hold (Zimper, 2011), that is

$$I[u(f)] = I[I_1[u(f)], ..., I_m[u(f)]] \tag{15}$$

or, equivalently, $I[u(f)] = I[u(\tilde{f})]$. It makes the model tractable with applications, thanks to the use of a folding back procedure. Obviously, the relation (15) is only valid when each E_j is f-convex. This condition is equivalent to comonotonicity of f with $\tilde{f}$.

Theorem 2. *Let $\{\succcurlyeq_j\}_{j=0,...,m}$ be a class of preference relations on $\mathcal{A}$ represented by $I_j[u(.)]$ for all $j = 0, ..., m$. Then the following statements are equivalent:*

(i) For any $f \in \mathcal{A}$, the law of iterated expectations holds when f is comonotonic with its conditional certainty equivalent act;

(ii) $\{\succcurlyeq_j\}_{j=0,...,m}$ satisfy f-convex dynamic consistency;

(iii) For any $f \in \mathcal{A}$ and any $j = 1, ..., m$ such that E_j is f-convex, $I_j[u(f)]$ uses the decision weight $p_j^f(.)$ given by:

$$p_j^f(A) = \frac{p(A \cap E_j)}{p(E_j)}, \quad A \in \Sigma \tag{16}$$

Proof **Part A.** $(i) \Rightarrow (ii)$. Let $f, g \in \mathcal{A}$ be such that $f \succcurlyeq g$. Assume that f and g differ only on event E_j, for any $j = 1, ..., m$. Then, under the law of iterated expectations, $f \succcurlyeq g$ if and only if $I[u(\tilde{f})] \geq I[u(\tilde{g})]$ hence, by monotonicity of $I[u(.)]$, $I_j[u(f)] \geq I_j[u(g)]$. Now we prove that each E_j is necessarily f-convex. W.l.o.g., let $\tilde{f}$ be ordered in the following way: $\tilde{f}_m \succcurlyeq ... \succcurlyeq \tilde{f}_1$. Assume that, for $s', s'' \in E_j$ and $s \in E_i$, we have $f(s') \succcurlyeq f(s) \succcurlyeq f(s'')$. Hence comonotonicity of f with $\tilde{f}$ entails that if $i > j$, then $f(s'') \succcurlyeq f(s)$, and if $i < j$ then $\tilde{f}(s) \succcurlyeq \tilde{f}(s')$. For strict preferences, we get a contradiction thus $i = j$. Then comonotonicity of f with $\tilde{f}$ implies $f(s') \succcurlyeq f(s) \succcurlyeq f(s'')$ only if $s \in E_j$.

Part B. $(ii) \Rightarrow (iii)$. The proof of part A of theorem 1 may be adapted with $E = E_j$ for each $j = 1, ..., m$.

Part C. $(iii) \Rightarrow (i)$. See Zimper (2011) and Lapied and Toquebeuf (2011). □

Finally, note that if we also assume consequentialism together with a convex range capacity, then one should obtain the result of Koida (2010): $\nu_j(.)$, for $j = 0, ..., m$, must exhibit an exponential form. It should be noted that the restriction of dynamic consistency in axiom 1 is analogous enough to the one proposed by Koida for his recursion axioms linking acts and two-stage acts. Indeed, "Reduction of two-stage acts" is only imposed on *nest-monotonic* acts, that are acts which are comonotonic with their conditional certainty equivalent act, in our terminology.

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