How to account for changes in the size of Sports Leagues:
The Iso Competitive Balance Curves

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1) Introduction.

The issue of Competitive Balance is a central issue in the literature on the economics of professional sports. The basic idea is that the managers of professional sports leagues must maintain a certain level of competitive balance in their league if they want it to remain attractive (Rottenberg (1956), El Hodiri & Quirk (1971), Fort & Quirk (1995), Vrooman (1995), Kesenne (2000), ...). An important part of literature is also devoted to the mechanisms to restore a satisfactory level of competitive balance: salary caps, luxury taxes, draft rules, gate revenue sharing… Although some authors challenge the idea that a decrease in competitive balance necessarily leads to a weakening of fan interest (Szymanski (2001)), all agree on the need to measure adequately the balance. As mentioned by Zymbalist (2002), the most commonly used index is the standard deviation of win percentage. But other indexes can be used as the ratio of the actual to the idealized standard deviation of win percentages, the Gini coefficient of win percentages, the Hirshman-Herfindahl index of competitive balance, the Concentration Ratio, the ratio of the top to bottom win percentages, the index of dissimilarity…

Analysis of the within-season competitive balance requires consideration of possible changes in the size of the league (that sometimes occur from one year to another). Indeed, indicators of competitive balance are sensitive to the number of teams comprising the league in the same way as indices measuring the degree of concentration in an industry are sensitive to the number of firms in the industry (Kamerschen & Lam (1975), Davies (1979)). As part of the analysis of competitive balance in professional sports, Depken (1999) and Pawlowki, Hovemann & Breuer (2010), have proposed a modified Herfindahl-Hirshman index to correct the measure of competitive balance depending on the size of the league. The change in size of a league is not a secondary issue in many professional sports in Europe and North America (see Table 1).

Table 1: Changes in the size of football leagues in Europe and North America

European Promotion/Relegation leagues (since 1960)	
French Premier League	1960 to 1963: 20 teams, 1964 and 1965: 18 teams, from 1966 to 1968: 20 teams, 1969 and 1970: 18 teams, from 1971 to 1997: 20 teams, from 1998 to 2002: 18 teams, from 2003 to 2011: 20 teams
Spanish Premier League	1960 to 1971: 16 teams, from 1972 to 1987: 18 teams, from 1988 to 1995: 20 teams, 1996 and 1997: 22 teams, from 1998 to 2011: 20 teams
English Premier League	1960 to 1987: 22 teams, 1988: 21 teams, from 1989 to 1991: 20 teams, from 1992 to 1995: 22 teams, from 1996 to 2011: 20 teams
Italian Premier league	1960 to 1967: 18 teams, from 1968 to 1988: 16 teams, from 1989 to 2004: 18 teams, from 2005 to 2011: 20 teams
German Premier League	1964 and 1965: 16 teams, from 1966 to 1991: 18 teams, 1992: 20 teams, from 1993 to 2011: 18 teams
North American closed league	
Major League Soccer	1996 to 1997: 10 teams, from 1998 to 2000: 12 teams, from 2001 to 2004: 10 teams, 2005 and 2006: 12 teams, 2007: 13 teams, 2008: 14 teams, 2009: 15 teams, 2010: 16 teams, 2011: 18 teams

Examining this question, Adjemian, Gayant & Le Pape (2012) have shown that the type of correction suggested by Depken (1999) and Pawlowki, Hovemann & Breuer (2010) is inadequate or incomplete in the sense that if it neutralizes the variability of the lower bound of the CB index on the size of the league, it doesn't take into account the variability of the upper bound. Then, they have proposed a Herfindahl-Hirshman Ratio of competitive balance whose lower and upper bounds are (i) insensitive to the size of the league (ii) respectively equal to 0 and 1. In the same vein, we propose in this note a robust competitive balance ratio constructed on the basis of a standard deviation of the percentage of points, and, by analogy with the work of Davies (1979) on the issue of industrial concentration, we construct Iso Competitive Balance curves. The note is organized as follows : in section 2, we discuss the importance of the point award system for a satisfying measure of Competitive Balance ; in section 3, we express a ratio of Competitive Balance derived from the standard deviation of the percentage of points ; finaly, in section 4, we construct Iso Competitive Balance curves.

2) Measure of Competitive Balance and point award system.

Since the objective is to build a robust and general measure of Competitive Balance, and since in some sports match results can be draws, we will focus on the calculation of the dispersion of the distribution of percentage of points rather than of winning percentage. Let us consider the case of a m-k-0 (m, k $\in \mathbb{N}$) point award system (m points for a win, k points for a draw, 0 point for a loss, with m > k). Depending on the value of k relatively to m, the total points awarded to all teams at the end of the season may not be constant but depends on the number of draws that occurred. More precisely, in a "once home-once away" league with N teams (N $\geq$ 2), if T ($0 \leq T \leq N(N-1)$) denotes the aggregate number of ties, the total number of points distributed during a season equals to $P_{m,k}(N) = (2k-m)T + mN(N-1)$. If, from one season to another, with the same number of teams in the league, $P_{m,k}(N)$ is not constant, the same percentage of points earned by a team can cover different degrees of Competitive Balance[1]. So, to construct a reliable index of competitive balance, it is needed that the total points distributed during the season are constant. This condition is fulfilled if and only if m = 2k points. If the actual allocation of points in the league does not respect this condition, one can not calculate an index of competitive balance on its basis despite we know that this point award system has an influence on the outcome of games and therefore on the actual level of competitive balance! This means that we can only measure the *ex post* degree of competitive balance of a championship in which the results are conditioned by a point award system that may *ex ante* have "altered" the degree of competitive balance.

3) Changes in size of the league and the maximum value of σ².

Let us denote by p_i the number of points obtained by the ith team (i $\in$ {1, …,N}) in a m-k-0 point award system. The percentage of point obtained by a team is $s_i = \dfrac{p_i}{\sum\limits_{i=1}^{N} p_i} = \dfrac{p_i}{P_{m,k}(N)}$. The

standard deviation of the percentage of points is $\sigma^2 = \dfrac{1}{N}\sum\limits_{i=1}^{N}(s_i - \bar{s})^2$ with $\bar{s} = \dfrac{1}{N}$. When the

size of the league is constant, this standard deviation is a good index to yearly measure the evolution of competitive balance. However, when the number N of teams comprising the league changes, it is essential to put into perspective the measured level of standard deviation

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[1] Such is the case in the actual European football leagues where m = 3 and k = 1.

with what would have been its maximum level. Indeed, the maximum level of deviation is dependent on N. We therefore propose to measure the competitive balance with the index CBR = $\dfrac{\sigma^2}{\sigma^2_{max}}$ (CBR for Competitive Balance Ratio). It is then necessary to determine the value of σ^2_{max} depending on the size of the league. For this purpose, Adjemian, Gayant & Le Pape (2012) have shown that the configuration of so-called "Perfect Hierarchy" is the one that maximizes the level of the standard deviation of percentage of points in any league of N teams. The configuration of Perfect Hierarchy can be described as follows (in a in a "once home-once away" championship) :

The 1st team wins its [2 × (N-1)] games, the 2nd team looses 2 games (the 2 games against the previous one) and wins [2 × (N-2)] games, the 3rd team looses 4 games (the 4 games against the 2 previous ones) and wins [2 × (N-3)] games, ..., the Nth team looses its [2 × (N-1)] games (No draw occurs in the championship).

Under Perfect Hierarchy, the number of points obtained by the ith team ($i \in [1; N]$) is p$_i$ = 2m(N-i) and its share of points is s$_i$ = $\dfrac{2m(N-i)}{mN(N-1)} = \dfrac{2(N-i)}{N(N-1)}$ (i $\in$ {1, ...,N}). The configuration of Perfect Hierarchy maximizes σ^2 because no marginal change of result in the championship is likely to increase the dispersion of the percentage of points. For example, any draw between two following teams (which is the smallest move away from this configuration) will decrease the dispersion of percentage of points. It is then easy to calculate the standard deviation of percentage of points $\sigma^2_{Perfect\ Hierarchy}$ = σ^2_{max} = $\dfrac{N+1}{3N^2(N-1)}$. Finally, we can express CBR = $\dfrac{\sigma^2}{\sigma^2_{max}} = \dfrac{3N^2(N-1)\sigma^2}{N+1}$.
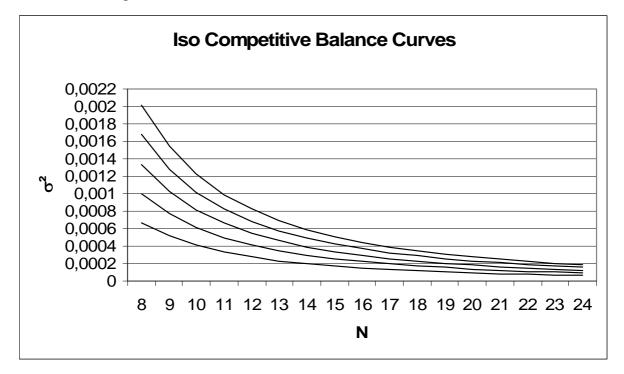
4) Iso Competitive Balance curves

In the literature on the measurement of industrial concentration, there is a historical concern: the ability of dissociating, on the one hand, the effect of the intrinsic inequality of market shares of firms, and, on the other hand, the effect of the number of firms in this industry. Davies (1979) proposes a comparison of many existing measures and constructs iso-concentration curves. Our work on the theme of the influence of the number of "firms" on the measured level of Competitive Balance has some similarities with the analysis carried out by

Davies. By extending our approach, we are brought to propose Iso Competitive Balance curves that indicate, for any value of N, the level of the standard deviation leading to a given level of Competitive Balance.

The equation of any curve Iso Competitive Balance is CBR = K and, by varying K between zero and one[2], we can represent a beam of Iso Competitive Balance curves, expressing that $\sigma^2 = \dfrac{(N+1)K}{3N^2(N-1)}$. In the chart below, the Iso Competitive Balance curves for a CBR between 0.1 and 0.3 are figured.

Iso Competitive Balance Curves

5) Conclusion

Since the article by Depken (1999), it is understood that it is necessary to correct the indices of competitive balance to reflect any changes in the size of the league. In this note, we propose Iso competitive balance curves to visualize the amount of correction to be made to ensure comparability between the observed levels of competitive balance in a league before and after changes in the number of teams.

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[2] Obviously, the case K = 1 corresponds to the configuration of Perfect Hierarchy

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